EDUMATE

Statistics

Government of Kerala
DEPARTMENT OF EDUCATION

State Council of Educational Research and Training (SCERT), Kerala
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Foreword

As part of the comprehensive revision of curriculum from pre-primary to the Higher Secondary sector, new textbooks have been developed for Std. XI and Std XII during the years 2014-15 and 2015-16 respectively. Evaluation activities should go hand in hand with the new curriculum. Real learning takes place by constructing knowledge through various learning processes.

In a constructive classroom, learners have opportunities to engage in a number of activities in which a range of attributes can be developed. The same activities provide the learner with scope for assessing development of these attributes. Hence there has been a shift from assessing only the products of learning to the process of learning. Anyhow it is to be noted that term end assessment is a part of continuous and comprehensive evaluation.

The main objective of this book is to help the learners to face the public examination with confidence. In this context, questions from all chapters of each subject of Std. XII have been developed along with the scoring indicators. Hope that this question bank titled “Edumate” will be helpful to learners as well as teachers. Your comments and suggestions are welcome and will assist us in improving the content of this book.

Wish you all the best.

Dr. J. Prasad
Director
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Learning Outcome

• Uses the Karl Pearson’s coefficient of correlation.

Q.1 The extreme values of the correlation coefficient is:
   a) 0 and 1   b) -1 and +1   c) -1 and 0   d) None of these
Score: 1, Time: 2 mts

⚠️ Scoring Indicators
Ans: b) -1 and +1 ................................................................. (1)

Learning Outcome

• Uses the Karl Pearson’s coefficient of correlation.

Q.2 If the correlation between the variables X and Y is 0.4, then the correlation between 3X and 2Y is:
   a) 0.24   b) 2.4   c) 0.4   d) 0.9
Score: 1, Time: 2 mts

⚠️ Scoring Indicators
Ans: c) 0.4 .................................................................................. (1)

Learning Outcome

• Recognises different types of correlation.

Q.3 The correlation between ‘time’ and ‘speed’ is:
   a) Positive   b) Negative   c) Zero   d) Mixed
Score: 1, Time: 2 mts

⚠️ Scoring Indicators
Ans: b) Negative ........................................................................ (1)
Learning Outcome

• Uses the Karl Pearson’s coefficient of correlation.

Q.4 The correlation between X+7 and Y-9 is -0.6, then the correlation between X and Y is:
   a) -0.6  b) 0.8  c) 0.6  d) 0

Score: 1, Time: 2 mts

Scoring Indicators

Ans: a) -0.6 ................................................................. (1)

Learning Outcome

• Identifies the meaning of correlation.

Q.5 We measure heights and weights of 100 twenty-year old male college students. Choose the correct answer.
   a) corr (height, weight) will be much greater than corr (weight, height)
   b) corr(weight, height) will be much greater than corr(height, weight)
   c) Both will have the same correlation.
   d) None of these.

Score:1, Time: 2 mts

Scoring Indicators

Ans: c) Both will have the same correlation ................................................. (1)

Learning Outcome

• Recognises different types of correlation.

Q.6 Identify the type of correlation present in the following cases.
   a) Distance from source and intensity of light
   b) Beauty and intelligence
   c) Height and intelligence
   d) Income and expenditure.

Score: 2, Time: 3 mts

Scoring Indicators

a) Negative correlation..............................................................................

b) Zero correlation ....................................................................................

c) Zero correlation ....................................................................................

d) Positive correlation
   Each carry ½ score.
**Learning Outcome**

- Uses rank correlation coefficient in suitable situations.

**Q.7** A ‘Pookkalam’ competition was conducted in connection with the Onam Festival in a higher secondary school. Two judges evaluated the performance of participants and assigned ranks as given below.

<table>
<thead>
<tr>
<th>Class</th>
<th>XII A</th>
<th>XI B</th>
<th>XI C</th>
<th>XI D</th>
<th>XI E</th>
<th>XII A</th>
<th>XII B</th>
<th>XII C</th>
<th>XII D</th>
<th>XII E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Judge A</td>
<td>9</td>
<td>7</td>
<td>8</td>
<td>1</td>
<td>6</td>
<td>5</td>
<td>19</td>
<td>3</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Judge B</td>
<td>1</td>
<td>6</td>
<td>9</td>
<td>3</td>
<td>5</td>
<td>8</td>
<td>4</td>
<td>7</td>
<td>10</td>
<td>12</td>
</tr>
</tbody>
</table>

a) Which correlation coefficient will be used to check whether two judges have the same approach?

b) Compute the correlation coefficient?

c) Are the two judges have the same outlook in assigning ranks?

Score: 5, Time: 8 mts

**Scoring Indicators**

- Rank correlation coefficient .................................................................................. (1)
- \[ \rho = 1 - \frac{6\sum d^2}{n^3 - n} \] ........................................................................... (1)
- Preparation of table of \( d^2 \) ................................................................................ (1)
- \( \rho = -0.57 \) (Making calculation) ........................................................................ (1)
- The judges have different approach. ....................................................................... (1)

**Learning Outcome**

- Uses the Karl Pearson’s coefficient of correlation.

**Q.8** Two separate tests are designed to measure a student’s ability to solve problems. Several students are randomly selected to take both tests and the results are:

<table>
<thead>
<tr>
<th>Test A(x)</th>
<th>43</th>
<th>65</th>
<th>73</th>
<th>34</th>
<th>99</th>
<th>78</th>
<th>65</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test B(y)</td>
<td>39</td>
<td>60</td>
<td>62</td>
<td>54</td>
<td>85</td>
<td>70</td>
<td>20</td>
</tr>
</tbody>
</table>

Calculate Karl Pearson’s correlation coefficient.

Score: 4, Time: 8 mts

**Scoring Indicators**

\[ r = \frac{n\sum xy - \sum x \sum y}{\sqrt{n\sum x^2 - (\sum x)^2} \sqrt{n\sum y^2 - (\sum y)^2}} \] .................................................. (1)
Preparing table to find
\[ \sum x = 457, \sum y = 390, \sum x^2 = 32669, \sum y^2 = 24406 \text{ and } \sum xy = 27114 \ldots \] (2)
\[ r = 0.6 \text{ (Making calculations)} \ldots \] (1)

**Learning Outcome**

- Uses rank correlation coefficient in suitable situations.

**Q.9** The marks obtained by 10 students in a written examination and their respective marks in the oral test are given below.

Mark in the written exam:
- 60
- 75
- 90
- 100
- 85
- 80
- 88
- 83
- 65
- 50

Ranks in the oral test:
- 8
- 7
- 2
- 1
- 6
- 5
- 4
- 9
- 10
- 3

Calculate the rank correlation coefficient.

Score: 4, Time: 8 mts

**Scoring Indicators**

\[ \rho = 1 - \frac{6 \sum d^2}{n^3 - n} \] (1)

Ranking the marks in written examination and preparation of the table of \( d^2 \ldots \) (2)
\[ \rho = 0.54 \text{ (Making calculation)} \ldots \] (1)

**Learning Outcome**

- Uses the Karl Pearson’s coefficient of correlation.

**Q.10** Using the marks awarded to 10 students in Economics and Accountancy, it is calculated that
\[ \sum x = 247, \sum y = 263, \sum x^2 = 7345, \sum y^2 = 7537 \text{ and } \sum xy = 7259 \ldots \]
Find the correlation coefficient.

Score: 3, Time: 5 mts

**Scoring Indicators**

\[ r = \frac{n \sum xy - \sum x \sum y}{\sqrt{n \sum x^2 - (\sum x)^2} \sqrt{n \sum y^2 - (\sum y)^2}} \] (1)

Making calculations \ldots (1)
\[ r = 0.87 \ldots \] (1)
Learning Outcome

- Uses rank correlation coefficient in suitable situations.

Q.11 The table below shows the marks awarded to six children in a competition by two judges. Calculate the coefficient of rank correlation.

<table>
<thead>
<tr>
<th>Child:</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Judge I:</td>
<td>6.8</td>
<td>7.3</td>
<td>8.1</td>
<td>9.8</td>
<td>7.1</td>
<td>9.2</td>
</tr>
<tr>
<td>Judge II:</td>
<td>7.8</td>
<td>9.4</td>
<td>7.5</td>
<td>9.6</td>
<td>8.9</td>
<td>6.9</td>
</tr>
</tbody>
</table>

Score: 4, Time: 7 mts

Scoring Indicators

\[
\rho = 1 - \frac{6}{n^2 - n} \sum d^2
\]

Ranking the marks and preparation of the table of \( d^2 \) .............................................. (2)

\[\rho = 0.09 \text{ (Making calculation)}\] .............................................. (1)

Learning Outcome

- Uses rank correlation coefficient in suitable situations.

Q.12 A doctor asked 10 of his patients, who were smokers, how many years they had smoked. He gave them a grade between 0 and 100 indicating the extent of their lung damage. The following table shows the result.

<table>
<thead>
<tr>
<th>Patient:</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of years</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Of smoking:</td>
<td>15</td>
<td>22</td>
<td>25</td>
<td>28</td>
<td>31</td>
<td>33</td>
<td>36</td>
<td>39</td>
<td>42</td>
<td>48</td>
</tr>
<tr>
<td>Lung damage</td>
<td>30</td>
<td>50</td>
<td>55</td>
<td>30</td>
<td>57</td>
<td>35</td>
<td>60</td>
<td>72</td>
<td>70</td>
<td>75</td>
</tr>
</tbody>
</table>

Calculate the coefficient of rank correlation between the number of years of smoking and the extent of lung damage.

Score: 4, Time: 8 mts

Scoring Indicators

\[
\rho = 1 - \frac{6}{n^2 - n} \sum (m^2 - m)
\]

\[\rho = \frac{6(\sum d^2 + CF)}{n^2 - n} \text{ where } CF = \frac{\sum (m^2 - m)}{12}
\]

Ranking the values and preparing the table of \( d^2 \) .............................................. (1½)

Here the rank 9.5 is repeated 2 times. So \( m=2 \) and \( CF=0.5 \) .............................................. (½)

\[\rho = 0.84 \text{ (Making calculations)}\] .............................................. (1)
Learning Outcomes

- Explains properties of regression coefficients.

Q.1 If the regression coefficient $b_{xy} = 1/2$, then which of the following values can be assumed by $b_{yx}$?

a) 4  
 b) 3  
 c) 5/2  
 d) 3/2

Score: 1, Time: 2 mts

⚠️ Scoring Indicators

d) 3/2  (1)

Learning Outcomes

- Explains properties of regression coefficients.

Q.2 The regression line of $Y$ on $X$ is $5x-7y=10$, then the regression coefficient of $Y$ on $X$ is:

a) 5/7  
 b) 7/5  
 c) -7/5  
 d) 10/7

Score: 1, Time: 2 mts

⚠️ Scoring Indicators

a) 5/7  ................................................................. (1)

Learning Outcomes

- Estimates unknown values for corresponding values given.

Q.3 To estimate the value of $X$ for a given value of $Y$, the regression equation used is:

a) $Y$ on $X$  
 b) $X$ on $Y$  
 c) both of these  
 d) none of these

Score: 1, Time: 1 mts

⚠️ Scoring Indicators

b) $X$ on $Y$ ................................................................. (1)
Learning Outcomes

- Estimates unknown values for corresponding values given.

Q4 A survey was conducted to study the relationship between daily expenditure on accommodation \((x)\) and expenditure on food and entertainment \((y)\) of sales representatives. The following results were obtained.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expenditure on accommodations ((x))</td>
<td>173</td>
<td>63.15</td>
</tr>
<tr>
<td>Expenditure on food and entertainment ((y))</td>
<td>47.8</td>
<td>22.98</td>
</tr>
<tr>
<td>Coefficient of correlation = +0.57</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. Write down the regression equation \(y\) on \(x\).

b. Estimate the expenditure on food if the expenditure of accommodation is Rs. 200.

Score: 4, Time: 8 mts

Scoring Indicators

a. Regression equation of \(y\) on \(x\) is

\[ y - \bar{y} = b_{yx} (x - \bar{x}) \]  

\[ b_{yx} = r \frac{\sigma_y}{\sigma_x} = 0.207 \]

The regression line is

\[ y = 0.207x + 11.99 \]

b. Expenditure of food when expenditure of accommodation \((x)\) is 200 is

\[ y = 0.207 \times 200 + 11.99 = 53.39 \]

Learning Outcomes

- Identifies the concept of regression analysis.
- Estimates unknown values for corresponding values given.

Q5 A biologist assumes that there is a linear relationship between the amount of fertilisers supplied to tomato plants and the subsequent yield of tomatoes obtained. Eight tomato plants of the same variety, were selected at random and treated, weekly with a solution in which \(x\) grams of fertiliser was dissolved in a fixed quantity of water. The yield \(y\) kgs of tomatoes was recorded.

<table>
<thead>
<tr>
<th>Plant</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
<td>2.5</td>
<td>3.0</td>
<td>3.5</td>
<td>4.0</td>
<td>4.5</td>
</tr>
<tr>
<td>Y</td>
<td>3.9</td>
<td>4.4</td>
<td>5.8</td>
<td>6.6</td>
<td>7.0</td>
<td>7.1</td>
<td>7.3</td>
<td>7.7</td>
</tr>
</tbody>
</table>

a) Obtain the regression line of \(y\) on \(x\).

b) Estimate the yield of a plant treated with 3.2 grams of fertiliser.

Score: 5, Time: 10 mts
Scoring Indicators

\[ \begin{array}{cccccc}
X & Y & XY & X^2 & Y^2 \\
1 & 3.9 & 3.9 & 1 & 15.21 \\
1.5 & 4.4 & 6.6 & 2.25 & 19.36 \\
2 & 5.8 & 11.6 & 4 & 33.64 \\
2.5 & 6.6 & 16.5 & 6.25 & 43.56 \\
3 & 7 & 21 & 9 & 49 \\
3.5 & 7.1 & 24.85 & 12.25 & 50.41 \\
4 & 7.3 & 29.2 & 16 & 53.29 \\
4.5 & 7.7 & 34.65 & 20.25 & 59.29 \\
\end{array} \]

\[ \sum X = 22 \quad \sum Y = 49.8 \quad \sum XY = 148.3 \quad \sum X^2 = 71 \quad \sum Y^2 = 323.76 \]

Preparation of the table. ................................................................. (1½)

The regression line of Y on X is

\[ y - \bar{y} = b_{yx} (x - \bar{x}) \]  ................................................................. (1)

\[ b_{yx} = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2} = 1.08, \bar{x} = 2.75, \bar{y} = 6.225 \]  ................................................................. (1)

The regression line is:

\[ y - 6.225 = 1.08(x - 2.75) \]  ................................................................. (½)

b) When \( x = 3.2, y = 6.711 \) ................................................................. (1)

Learning Outcomes

- Identifies the concept of regression analysis.
- Estimates unknown values for corresponding values given.

Q.6 The following table exhibits sales (in thousands of rupees) and the number of sales persons employed for different years.

<table>
<thead>
<tr>
<th>Sales</th>
<th>120</th>
<th>125</th>
<th>118</th>
<th>115</th>
<th>100</th>
<th>130</th>
<th>140</th>
<th>135</th>
<th>130</th>
<th>123</th>
</tr>
</thead>
<tbody>
<tr>
<td>No of persons</td>
<td>10</td>
<td>15</td>
<td>12</td>
<td>18</td>
<td>20</td>
<td>21</td>
<td>22</td>
<td>20</td>
<td>15</td>
<td>19</td>
</tr>
</tbody>
</table>

Develop a regression line to predict the sales based on the number of sales persons employed.

Score: 4, Time: 8 mts

Scoring Indicators

Let X denotes the sales and Y denotes the number persons employed. We have to find the regression line of X on Y.
The regression line of X on Y is: \[ x - \bar{x} = b_{yx} (y - \bar{y}) \] ............................................. (1)

Preparation of the table. .......................................................... (1\%)

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>XY</th>
<th>Y²</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>10</td>
<td>1200</td>
<td>100</td>
</tr>
<tr>
<td>125</td>
<td>15</td>
<td>1875</td>
<td>225</td>
</tr>
<tr>
<td>118</td>
<td>12</td>
<td>1416</td>
<td>144</td>
</tr>
<tr>
<td>115</td>
<td>18</td>
<td>2070</td>
<td>324</td>
</tr>
<tr>
<td>100</td>
<td>20</td>
<td>2000</td>
<td>400</td>
</tr>
<tr>
<td>130</td>
<td>21</td>
<td>2730</td>
<td>441</td>
</tr>
<tr>
<td>140</td>
<td>22</td>
<td>3080</td>
<td>484</td>
</tr>
<tr>
<td>135</td>
<td>20</td>
<td>2700</td>
<td>400</td>
</tr>
<tr>
<td>130</td>
<td>15</td>
<td>1950</td>
<td>225</td>
</tr>
<tr>
<td>123</td>
<td>19</td>
<td>2337</td>
<td>361</td>
</tr>
</tbody>
</table>

\[ \sum X = 1236 \quad \sum Y' = 172 \quad \sum X'Y' = 21358 \quad \sum Y'^{2} = 3104 \]

\[ b_{yx} = \frac{n \sum xy - \sum x \sum y}{n \sum x^{2} - (\sum x)^{2}} = 0.68, \bar{x} = 123.6, \bar{y} = 17.2 \] ............................................. (1)

The regression line is: \[ x - 123.6 = 0.68(y - 17.2) \] ............................................. (1\%)

---

**Learning Outcomes**

- Identifies the concept of regression analysis.
- Estimates unknown values for corresponding values given.

**Q.7** A large supermarket adopted a new strategy to increase its sales. It has adopted a few consumer friendly policies and is using video clips of 15 minutes to propagate the new policies. The following table provides the data about the number of video clips shown in randomly selected day and the sales turnover in the corresponding day.

<table>
<thead>
<tr>
<th>Day</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>No of video clips</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>35</td>
<td>35</td>
<td>35</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>50</td>
</tr>
<tr>
<td>Sales turnover</td>
<td>150</td>
<td>210</td>
<td>140</td>
<td>180</td>
<td>230</td>
<td>270</td>
<td>310</td>
<td>330</td>
<td>300</td>
<td>270</td>
</tr>
</tbody>
</table>

(in thousands of Rs)

a) Develop the regression line to predict the sales from the number of video clips shown.

b) Estimate the sales when 60 video clips are shown.

Score: 5, Time: 10 mts
Scoring Indicators

a) Let X denotes the number of video clips and Y denotes the sales in thousands of Rs. We have to find the regression line of Y on X.

The regression line of Y on X is: \[ y - \bar{y} = b_{yx}(x - \bar{x}) \] ........................................ (1)

Preparation of the table. ................................................................. (1\%)

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>XY</th>
<th>X^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>150</td>
<td>3750</td>
<td>625</td>
</tr>
<tr>
<td>25</td>
<td>210</td>
<td>5250</td>
<td>625</td>
</tr>
<tr>
<td>25</td>
<td>140</td>
<td>3500</td>
<td>625</td>
</tr>
<tr>
<td>35</td>
<td>180</td>
<td>6300</td>
<td>1225</td>
</tr>
<tr>
<td>35</td>
<td>230</td>
<td>8050</td>
<td>1225</td>
</tr>
<tr>
<td>35</td>
<td>270</td>
<td>9450</td>
<td>1225</td>
</tr>
<tr>
<td>40</td>
<td>310</td>
<td>12400</td>
<td>1600</td>
</tr>
<tr>
<td>40</td>
<td>330</td>
<td>13200</td>
<td>1600</td>
</tr>
<tr>
<td>40</td>
<td>300</td>
<td>12000</td>
<td>1600</td>
</tr>
<tr>
<td>50</td>
<td>270</td>
<td>13500</td>
<td>2500</td>
</tr>
</tbody>
</table>

\[ \sum X = 350 \] \[ \sum Y = 2390 \] \[ \sum XY = 87400 \] \[ \sum X^2 = 12850 \]

\[ b_{yx} = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2} = 6.25, \bar{x} = 3.5, \bar{y} = 23.9 \] ........................................ (1)

The regression line is:

\[ y - 23.9 = 6.25(x - 35) \] ..................................................... (1\%)}

b) When x = 60, y = 395.25 ................................................................. (1)

Learning Outcomes

- Recognises regression lines and their point of intersection.
- Explains properties of regression coefficients.

Q.8 A regression analysis on the income in thousands of Rs. (X) and expenditure in thousands of Rs. (Y) resulted in the following regression equations.

\[ x+2y - 5 = 0 \text{ and } 2x + 3y - 8 = 0 \]

a) Identify the regression equations

b) Compute the correlation coefficient.

c) Find the mean values of X and Y.

Score: 4, Time: 8 mts
Scoring Indicators

a) \[ x + 2y - 5 = 0 \] \hspace{1cm} \text{equation (1)}
\[ 2x + 3y - 8 = 0 \] \hspace{1cm} \text{equation (2)}

Let us assume equation (1) as the regression equation of \( Y \) on \( X \) and equation (2) as regression equation of \( X \) on \( Y \).

Let equation (1) be written as \[ y = -\frac{1}{2} x + \frac{5}{2} \Rightarrow b_{xy} = -\frac{1}{2} \]

Equation (2) as \[ x = -\frac{3}{2} y + \frac{8}{2} \Rightarrow b_{yx} = -\frac{3}{2} \]

\[ b_{yx} \times b_{xy} = 0.75 \] which is lesser than 1

Our supposition is right. That means (1) is regression equation of \( Y \) on \( X \) and (2) is regression equation of \( X \) on \( Y \).

b) \[ r = \sqrt{b_{yx} \times b_{xy}} = \sqrt{0.75} = 0.87 \] \hspace{1cm} \text{equation (1)}

c) Since the regression lines pass through the point \( (\bar{x}, \bar{y}) \), equations (1) and (2) become

\[ \bar{x} + 2\bar{y} - 5 = 0 \] \hspace{1cm} \text{equation (3)}
\[ 2\bar{x} + 3\bar{y} - 8 = 0 \] \hspace{1cm} \text{equation (4)}

Solving (3) and (4) we get the mean values \( \bar{x} = -2 \) and \( \bar{y} = 3.5 \).
Learning outcomes

- Recognises and uses the concept of first order and second order derivatives.
- Use concept of calculus in statistics.

Q.1 Choose the correct answer

(a) \( \frac{d}{dx} \left( \frac{1}{x} \right) = \ldots \)

(i) \(-9x^{-9}\)  (ii) \(-9x^{10}\)  (iii) \(\frac{-9}{x^{10}}\)  (iv) \(-9x^8\)

(b) The cost of manufacturing \(x\) items is given by \(c(x) = 2x^2 - 16x + 100\). To have a minimum cost find how many items are to be manufactured.

Time: 8 mnts, Score: 1+4

Scoring Indicators

(a) \(\frac{-9}{x^{10}}\) ........................................................... (1)

(b) For the \(c(x)\) to be minimum .................................................. (1)

\(C''(x) = 0\) and \(C''(x) > 0\) .................................................. (1)

\(C''(x) = 4x - 16\), \(C''(x) = 4\) ............................................. (½)

\(C''(x) = 0 \Rightarrow x = 4\) .................................................. (½)

At \(x = 4\), \(C''(x) > 0\) .................................................. (½)

Minimum cost is at \(x = 4\) .................................................. (½)

Minimum cost is given by .................................................. (½)

\(2 \times 4^2 - 16 \times 4 + 100 = 68\) .................................................. (1)

Learning outcomes

- Explains the concept of integration. Use definite integrals in suitable situations.

Q.2

(a) If \(\int x^k \, dx = \frac{x^{k+1}}{k+1} + c\), find \(k\).

(i) 6  (ii) 7  (iii) 1  (iv) 5

(b) Evaluate the integral \(\int_0^2 (x^2 - 3x + 4) \, dx\).

Time: 8 mnts, Score: 1+3
**Scoring Indicators**

(a) \( k=5 \) ................................................................. (1)

(b) Integrating the function ......................................................... (1)

  Applying the limits ................................................................. (1)

  Simplification ........................................................................ (1)

---

**Learning outcomes**

- Recognises and uses the concept of first order and second order derivatives. Explains the concept of integration.

Q.3 (a) Match the following

<table>
<thead>
<tr>
<th>( f(x) )</th>
<th>( f'(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x^3 )</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>3( x^2 )</td>
</tr>
<tr>
<td>( \frac{1}{x^2} )</td>
<td>0</td>
</tr>
<tr>
<td>3( x + 2 )</td>
<td>(-3x^-4)</td>
</tr>
</tbody>
</table>

(b) Find the value of \( C \) if \( \int_0^c x^2 \, dx = \frac{8}{3} \)

Time: 7 mnts, Score: 1+3

---

**Scoring Indicators**

(a) Match the following (1/2) each ......................................................... (2)

(b) Integrating the function ......................................................... (1)

  Applying the limits ................................................................. (1)

  Simplification, \( c = 2 \) ................................................................. (1)

---

**Learning outcomes**

- Recognises and uses the concept of first order and second order derivatives.

- Use definite integrals in suitable situations.

Q.4

(a) \( \int_{-1}^{1} (x+1) \, dx = \) ............

(b) If \( \frac{df(x)}{dx} = 4x^2 - 3 \) then find \( f''(0) \).

Time: 5 mnts, Score: 1+2
Scoring Indicators

(a) applying the property of definite integral .................................................. (1)
(b) \( f'(x) = 8x \) ............................................................................... (1)
\( f''(x) = 8 \) ....................................................................... (½)
\( f'''(0) = 8 \) ....................................................................... (½)

Learning outcomes

- Recognises and uses the concept of first order and second order derivatives

Q.5 A company sells one of its products for Rs.63 per unit. If \( x \) is the number of output of the product, the cost function for the same is \( \frac{x^3}{3} - \frac{3x^2}{2} - 7x \). Find the number of output of the product that maximizes the profit.

Time: 9 mnts, Score: 5

Scoring Indicators

\( R(x) = 63x \) ............................................................................... (1)
\( C(x) = \frac{x^3}{3} - \frac{33x^2}{2} - 7x \)
\( P(x) = R(x) - C(x) \) ....................................................................... (½)
\( P'(x) = \) ............................................................................... (1)
\( P''(x) = \) ....................................................................... (½)

For the \( P(x) \) is to be maximum

\( P'(x) = 0 \) and \( P''(x) < 0 \) ................................................................. (1)
\( P'(x) = 0 \) gives, \( x = \) ................................................................... (½)

Number of output that maximize the profit = .................................................. (½)

Learning outcomes

- Recognises and uses the concept of first order and second order derivatives

Q.6 If profit function of a firm \( P(x) = 70 + 30x - 30x^2 \) then

a) \( P'(x) = - - - - - \)  b) \( P'(x) = 0 \) gives, \( x = - - - \)
c) \( P''(x) = - - - - - \)  d) \( P(x) \) is maximum at \( x = - - - - \)

Time: 5 mnts, Score: 3

Scoring Indicators

a. \( P'(x) = 30 - 60x \) ............................................................................... (1)
b. \( P'(x) = 0 \) gives, \( x = 2 \) ................................................................... (½)
c. \( P''(x) = -60 \) ....................................................................... (½)
d. Applying the condition for maximum, \( P(x) \) is maximum at \( x = 2 \) .................................. (1)
Learning outcomes

- Recognises and uses the concept of first order and second order derivatives

Q.7  a) Mark the maximum and minimum of the function \( f(x) \) whose graph is given.

b) The cost function of Bronze Sculptor manufacturing is given by where \( x \) is the number of units produced.
   i) Write the conditions for \( C(x) \) to be minimum.
   ii) How many units are to be produced to have a minimum cost?
   iii) Find the minimum cost.

Time: 10 mnts, Score: 1+4

Scoring Indicators

a) Identifies the maximum and minimum points from the picture .......................... (1)

b) i) For the \( C(x) \) is to be minimum
   \[ C''(x) = 0 \quad \text{and} \quad C''(x) > 0 \] ............................................................ (1)
   ii) \( C(x) = 3x^2 - 6x + 15 \)
   \[ C''(x) = 6x - 6 \] ............................................................................. (1)
   \[ C''(x) = 0 \Rightarrow x = 1 \] ............................................................................. (½)
   \[ C''(x) = 6 \] ............................................................................. (½)
   iii) Minimum cost is given by \( 3 \times 1 - 6 + 15 = 12 \) .............................................. (1)

Learning outcomes

- Recognises and uses the concept of first order and second order derivatives

Q.8  Every business firm want to maximise their profit. An Ayurvedic Pharmacy produces 400 bottles of Chyavanaprasham in a batch. The profit function is given by \( \frac{-x^2}{400} + 2x - 80 \).

a) How will you use differential calculus to find the number of units to be produced for having maximum profit?

b) To have the maximum profit, is there any need of change in the Pharmacy’s present strategy?

Time: 9 mnts, Score: 1+3
Scoring Indicators

a) Identifies the problem and applies differential calculus.
   For maximum profit, \( x \) is such that derivative of \( P(x) \)
   i.e., \( P'(x) = 0 \) and \( P''(x) < 0 \), (states condition) ................................. (1)
   \[ P(x) = \frac{-x^2}{400} + 2x - 80 \]
   Finds derivative of \( P(x) \)
   \[ P'(x) = 0 \Rightarrow \frac{-x}{200} + 2 = 0 \] ......................................................... (1)
   \[ \frac{x}{200} = 2 \] ........................................................................................................... (½)
   \[ x = 400 \] ....................................................................................................................... (½)
   \[ P''(x) = \frac{-1}{200} < 0 \] .................................................................................................. (½)
   Hence the company gets the maximum profit when they produce 400 bottles per batch.
   Therefore there is no need of changing the present strategy. ........................................... (½)

Learning outcomes

- Recognises and uses the concept of first order and second order derivatives

Q.9 The revenue received on the sale of \( x \) books per day by MJS Publishers
   is given by \( 125x - \frac{5x^2}{3} \). The cost function is given by \( 500 + 13x + 0.2x^2 \).
   a) Write the profit function.
   b) Find how many books have to be sold to have the maximum profit.
   c) Calculate the maximum profit.

Time: 9 mnt, Score: 1+3+1

Scoring Indicators

\[ R(x) = 125x - \frac{5x^2}{3} \]
\[ C(x) = 500 + 13x + 0.2x^2 \]

a) \[ P(x) = R(x) - C(x) \] ................................................................. (½)
   \[ = 125x - \frac{5x^2}{3} - 500 - 13x - 0.2x^2 \]
   \[ = 112 - \frac{283}{15} x^2 - 500 \] ......................................................... (½)

b) Applies differentiation in maximisation

Now \[ P'(x) = \frac{112 - \frac{56}{15} x}{x} \] .............................................. (1)
   \[ P'(x) = 0 \Rightarrow \frac{56}{15} x = 112 \] ................................................................. (½)
   \[ x = 30 \] ...................................................................................................................... (½)
   \[ P''(x) = \frac{-56}{15} \] is negative. ................................................................. (½)
Therefore they have to sell 30 books per day. .................................................. (½)

c) Evaluates maximum profit

Maximum profit is $112 \times 30 - (30)^2 - 500$ ................................................. (½)

$= 1180 - ......................................................................................... (½)

**Learning outcomes**

- Use definite integrals in suitable situations.

**Q.10** The marginal cost of a company is given by $MC(x) = 3x^2 - 630x + 27000$. Find the cost function.

**Time: 7 mnts, Score: 3**

**Scoring Indicators**

The cost function is given by integral of marginal cost function......................... (1)

\[
C(x) = \int (3x^2 - 630x + 27000)dx = \frac{3}{3}x^3 - 630 \frac{x^2}{2} + 27000x + c \]

$= x^3 - 315x^2 + 27000x + c ......................................................... (½)$

**Learning outcomes**

- Identifies the concept of differentiation. Use definite integrals in suitable situations.

**Q.11** (a) Derivative of $x^n$ is $nx^{n-1}$. Find the value of $n$.

(b) Find the value of $k$, if $\int kx^6dx = 28x^6$

**Time: 6 mnts, Score: 1+2**

**Scoring Indicators**

(a) $\frac{n}{k} = 9$ ......................................................................................... (1)

(b) $\frac{6}{k} = 28$ ......................................................................................... (1)

$k = 168$ ......................................................................................... (1)
Learning Outcome

- Differentiates discrete and continuous random variables.

Q.1 Let $X$ is a random variable representing the number of heads obtained when a coin is tossed 6 times. The maximum value of $X$ is ........
   a) 1  b) 6  c) 5  d) 0

Score : 1, Time : 1 Minute

⚠️ Scoring Indicators
   b) 6 ........................................................................................................ (1)

Learning Outcome

- Recognises discrete and continuous probability distributions.

Q.2 The number of trees in a plot is an example of ........ variable
   a) qualitative  b) continuous  c) skewed  d) discrete

Score : 1, Time : 1 Minute

⚠️ Scoring Indicators
   d) Discrete ................................................................................................... (1)

Learning Outcome

- Recognises discrete and continuous probability distributions.

Q.3 The height of students in a school is ........ variable
   a) qualitative  b) continuous  c) skewed  d) discrete

Score : 1, Time : 1 Minute

⚠️ Scoring Indicators
   b) Continuous .............................................................................................. (1)
Learning Outcome

- Explains mathematical expectation of random variables.

Q.4 If \( E(X) = 7 \) \( E(Y) = -4 \), then \( E(X-Y) = \) …………..

a) 7  b) 11  c) 3  d) 0

Score: 1, Time: 1 Minute

⚠️ Scoring Indicators

b) 11 .......................................................................................................................... (1)

Learning Outcome

- Recognises discrete and continuous probability distributions.

Q.5 \( f(x) = 2^x, 0 \leq x \leq 1 \), then \( f(x) \) is …………

a) discrete probability function
b) continuous probability function
c) discrete RV
d) continuous RV

Score: 1, Time: 1 Minute

⚠️ Scoring Indicators

b) Continuous probability function................................................................. (1)

Learning Outcome

- Recognises discrete and continuous probability distributions.

Q.6 State whether the statement is true or false
If \( X \) and \( Y \) are two Random variables then \( X/Y \) is a Random variable \( Y \neq 0 \)

Score: 1, Time: 1 Minute

⚠️ Scoring Indicators

True ......................................................................................................................... (1)

Learning Outcome

- Differentiates discrete and continuous random variables.

Q.7 Let \( X \) be a discrete random variable with the following pmf

\[ P(x) = \begin{cases} 0.1 & \text{for } x=0.2 \\ 0.2 & \text{for } x=0.4 \end{cases} \]
\[
\begin{align*}
\text{for } x &= 0.5 \\
\text{for } x &= 0.8 \\
\text{for } x &= 1 \\
\text{otherwise}
\end{align*}
\]

Find
(a) Range of random variable \( X \)
(b) \( P(X \leq 0.5) \)
(c) \( P(0.25 \leq X \leq 0.75) \)

Score: 3, Time: 7 Minutes

**Scoring Indicators**

(a) \( \text{Range}=\{0.2,0.4,0.5,0.8,1\} \) ................................................................. (1)
(b) \( P(X \leq 0.5) = P(X = 0.2) + P(X = 0.4) + P(X = 0.5) \) ............................ \( \frac{1}{2} \)
   \[= P(0.2)+P(0.4)+P(0.5)\]
   \[= 0.1 + 0.2 + 0.2 = 0.5 \] ................................. \( \frac{1}{2} \)

**Learning Outcome**

- Recognises discrete and continuous probability distributions.

Q.8 Let \( X \) be the number of defective items when 2 items have shipped and \( P(x) \) be probability of \( x \) defective items. Use the table below to find \( P(X=2) \).

<table>
<thead>
<tr>
<th>( X )</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(X) )</td>
<td>( \frac{2}{5} )</td>
<td>( \frac{8}{15} )</td>
<td>-----</td>
</tr>
</tbody>
</table>

Score: 3, Time: 6 Minutes

**Scoring Indicators**

We know that ................................................................. (1)

\[
\begin{align*}
P(X = 0) + P(X = 1) + P(X = 2) &= 1 \\
\frac{2}{5} + \frac{8}{15} + P(X = 2) &= 1 \\
\frac{14}{15} + P(X = 2) &= 1 \\
P(X = 2) &= 1 - \frac{14}{15} = \frac{1}{15} \end{align*}
\] ................................. (2)
Learning Outcome

- Recognises discrete and continuous probability distributions.

Q.9  
(a) Write down the properties of a cumulative density function (cdf).
(b) If the cdf of a continuous random variable is,
\[ F(x) = \begin{cases} 
0, & \text{if } x \leq 0 \\
x^2, & \text{if } 0 \leq x \leq 1 \\
1, & \text{if } x \geq 1 
\end{cases} , \text{ find pdf.} \]

Score: 4, Time: 8 Minutes

⚠️ Scoring Indicators
- Properties of cdf ................................................................. (2)
- Determination of pdf .......................................................... (2)

Learning Outcome

- Evaluates mean and variance of random variables

Q.10  
A continuous random variable X has probability function f(x)
\[ f(x) = \begin{cases} 
2x, & \text{if } 0 \leq x \leq 1 \\
0, & \text{otherwise} 
\end{cases} \]

Find (a) Expected value of X    (b) \( E(1 - 2X) \)

Score: 4, Time: 8 Minutes

⚠️ Scoring Indicators
- \( E(X) = \int xf(x)dx \) ......................................................... (1)
- Calculation ........................................................................ (1)
- \( E(1 - 2X) = 1 - 2E(X) \) ............................................. (1)
- Calculation ........................................................................ (1)

Learning Outcome

- Recognises discrete and continuous probability distributions.

Q.11  
Let \( f(x) = \begin{cases} 
kr^2, & \text{if } 0 \leq x \leq 1 \\
0, & \text{otherwise} 
\end{cases} \), be a probability density function of a random variable. Find the value of K.

Score: 2, Time: 4 Minutes
Scoring Indicators

Value of k ........................................................................................................... (2)

Learning Outcome

- Evaluates mean and variance of random variables

Q.12 Let \( X \) has a p.d.f. \( f(x) = \begin{cases} \frac{1}{4}, & -2 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases} \).

Find cumulative density function of \( X \)

Score : 3, Time : 5 Minutes

Scoring Indicators

Cdf when \( x < -2 \) ............................................................................................. (1)
Cdf when \(-2 < x < 2 \) ............................................................................................ (1)
Cdf when \( x > 2 \) ................................................................................................. (1)

Learning Outcome

- Evaluates mean and variance of random variables

Q.13 (a) Variance of a random variable \( X \) is 16, then \( V(X+5) = \) ...........

(i) 0  (ii) 21  (iii) 256  (d) 16

(b) If probability mass function of a random variable \( X \) is given by

\[ P(x) = \begin{cases} \frac{x}{6}, & x = 1, 2, 3 \\ 0, & \text{otherwise} \end{cases}. \]

Determine (i) \( E(X) \)  (ii) \( E(2X+5) \)

Score : 4, Time : 8 Minutes

Scoring Indicators

a) 16 ............................................................................................................ (1)
b) \( E(X) \) ................................................................................................. (2)
\( E(2X+5) \) ............................................................................................... (1)

Learning Outcome

- Recognises discrete and continuous probability distributions.

Q.14 Examine whether \( f(x) = 5x^4, \ 0 < x < 1 \) can be a p.d.f of a continuous random variable \( X \).

Score : 3, Time : 5 Minutes
Scoring Indicators

Total probability = 1 ................................................................. (2)
Conclusion ....................................................................................... (1)

Learning Outcome

• Evaluates mean and variance of random variables

Q.15 Let Mean of $X$ is $\mu$ and variance of $X$ is $\sigma^2$ then

(a) $E(2X+3) =$ -------
(b) $V(1-2X) =$ -------

Score : 2, Time : 5 Minutes

Scoring Indicators

$E(2X+3)$ .................................................................................. (1)
$V(1-2X)$ .................................................................................. (1)

Learning Outcome

• Evaluates mean and variance of random variables

Q.16 Random variable $X$ has following pdf $f(x) = 2x$, $0 \leq x \leq 1$.

Find (a) $P(0 \leq X \leq 1)$ (b) $P\left(X \leq \frac{1}{2}\right)$ (c) $P\left(|X| \geq 1\right)$

Score : 2, Time : 10 Minutes

Scoring Indicators

a) $P(0 \leq X \leq 1)$ ....................................................................... (2)
b) $P\left(X \leq \frac{1}{2}\right)$ ................................................................... (2)
c) $P\left(|X| \geq 1\right)$ .................................................................. (1)

Learning Outcome

• Evaluates mean and variance of random variables

Q.17 a) If $F(X)$ is a cumulative density function of a continuous random variable $X$ then

$F(-\infty) =$ ............... 
(b) Find the constant $k$ such that $f(x) = \begin{cases} \frac{kx^2}{2}; & 0 < x < 3 \\ 0; & \text{otherwise} \end{cases}$ is a pdf

Score : 4, Time : 10 Minutes

Scoring Indicators

$F(-\infty) = 0$ ................................................................................ (1)
Calculation of $k$ ............................................................................. (3)
Learning outcomes

- Evaluates the probabilities by applying Poisson distribution.

Q.1 For a Poisson variable X, P(X=1) = P(X=2).

(a) Find the mean of X  
(b) Calculate P(X<1)

Time: 7 mnts, Score: 2+2

Scoring Indicators

(a) Pdf of poisson density function .................................................. (1)

Mean of x ................................................................................. (1)

(b) P(X<1)=P(X=0) .................................................................. (½)

Calculating probability .................................................................. (1½)

Learning outcomes

- Evaluates the probabilities by applying Binomial distribution.

Q.2 Random variable X has the following density function

\[ f(x) = nC_x \left( \frac{1}{3} \right)^x \left( \frac{2}{3} \right)^{n-x}, x = 0,1,2,\ldots,5 \]

Find  
(a) Values of parameters of the distribution

(b) Mean and variance of the distribution

Time: 6 mnts, Score: 1+2

Scoring Indicators

(a) comparing with binomial pdf

Finding n and p ................................................................. \( \left( \frac{1}{2} + \frac{1}{2} \right) \)

(b) Finding mean .................................................. (1)

Variance ............................................................................... (1)
Learning outcomes

- Evaluates the probabilities by applying Binomial distribution.

Q.3 If X follows binomial distribution with mean 18 and variance 6 then Probability of ‘success’, p is

(a) 3  (b) 12  (c) \( \frac{2}{3} \)  (d) \( \frac{1}{3} \)

Time: 2 mnts, Score: 1

**Scoring Indicators**

\[ p = \frac{2}{3} \]  

(1)

------------------------

Learning outcomes

- Evaluates the probabilities by applying Poisson distribution.

Q.4 (a) A random variable X follows a Poisson distribution with a standard deviation of 4. What is the expected value of X?

a. 9  b. 3  c. 16  d. 4

(b) If 3% of electric bulbs produced by a company are defective, find the probability that in a sample of 100 bulbs, exactly 5 bulbs are defective.

Time: 7 mnts, Score: 1+3

**Scoring Indicators**

(a) (c) ................................................................. (1)

(b) calculate \( \lambda = np = 100 \times 3 \% = 3 \) ................................................................. (1)

pdf of poisson distribution ................................................................. (1)

finding \( P(X=5) \)

------------------------

Learning outcomes

- Evaluates the probabilities by applying Poisson distribution.

Q.5 (a) Which of the following cannot be modelled by a Poisson distribution?

i) The weight of the tiffin box bought by a student.

ii) The number of telephone calls received by a switchboard in an hour

iii) The number of customers arriving at a petrol station on Christmas day

iv) The number of bacteria found per square metre of soil

(b) Assume that it is known that 80% of monkeys treated with a specific antibiotic recover from a particular disease. If 5 monkeys are treated, find the probability that at least 4 monkeys recover.

Time: 7 mnts, Score: 1+3
Scoring Indicators

(a) (i) ................. (1)
(b) identifying problem as an application of binomial distribution and writing its pdf ..... (1)
P=0.80 and q=0.20, n=5 ................................................................. (1)
P(X ≥ 4) = 1−P(X<4) ................................................................. (½)
= 0.737 .............................................................................. (1½)

Learning outcomes

- Evaluates the probabilities by applying Binomial distribution.

Q.6 (a) Which of the following is not a characteristic of a Binomial distribution?
   i) There is a sequence of identical trials
   ii) The trials are independent of one another
   iii) Each trial results in two or more outcomes
   iv) The probability of success (p) is the same for all trials

(b) At a wholesale protea nursery exactly 100 seeds are planted in each seed-bed, and the probability that a protea seed will germinate is 0.8. Find the expected number of seeds in the seed-bed that will germinate?

Time: 8 mnts, Score: 1+2

Scoring Indicators

(a) iii ........................................................ ........................................... (1)
(b) identifying n and p as n=100, p=0.8 ................................................ (1)
   Expected number of seeds = np = 0.8 x 100=80 ................................... (1)

Learning outcomes

- Defines Binomial distribution and Poisson distribution.

Q.7 Forty percent of the passengers who fly on a certain route do not check in any luggage. The planes on this route seat 18 passengers. For a full flight, what is the variance of the number of passengers who do not check in any luggage?
   a. 3.60          b. 3.68          c. 4.32          d. 3.75

Time: 2 mnts, Score: 1

Scoring Indicators

  c ........................................................ ........................................... (1)
Learning outcomes

- Evaluates the probabilities by applying Poisson distribution.

Q.8 A computer that operates continuously breaks down randomly on average 6 times per month. What is the probability of exactly 4 breakdowns in the first month?

a. 0.168  
   b. 0.134  
   c. 0.815  
   d. 0.285

Time: 2 mnts, Score: 1

Scoring Indicators

b. .......................................................... ................................................................. (1)

Learning outcomes

- Defines Binomial distribution and Poisson distribution.

Q.9 “Poisson distribution can be used to approximate the Binomial distribution”. Comment on this statement.

Time: 2 mnts, Score: 2

Scoring Indicators

Justifies the statement.

\[ n \to \infty \quad \text{..........................................................} \quad (1/2) \]

\[ p \to 0 \quad \text{..........................................................} \quad (1/2) \]

and \( np = \lambda \) is a finite constant.......................................................... (1)

Learning outcomes

- Evaluates the probabilities by applying Poisson distribution

Q.10 The average deaths due to snake bites in a village is 2. Let X be number of deaths due to snake bites in a year in the district.

a) Identify the distribution.

b) Write the p.d.f of the distribution

c) What is the probability of the number of snake bite deaths is one?

Time: 6 mnts, Score: 4

Scoring Indicators

a) Identifies the distribution - Poisson .......................................................... (1)

b) Recalls the pdf \( P(X=x) \) and find \( \lambda = 2 \) ........................................... (1/2+1/2)

c) Find \( P(X=1) \) using the pdf .......................................................... (2)
Learning outcomes

- Defines Binomial distribution and Poisson distribution.

Q.11 State which of the following statement is true or false.

i) In a binomial distribution mean = 5.2, variance = 6.5

ii) In a Poisson distribution mean = 6.5, variance = 5.2

iii) Binomial distribution tends to Poisson when n is large, probability
    \( p \to 0 \) and \( np \) is infinite.

iv) Binomial distribution is symmetric if \( p = 0.5 \)

Time: 2 mnts, Score: 2

Arrow Scoring Indicators

(i) false

(ii) false

(iii) false

(iv) true ................................................................. (1/2 x 4=2)
Learning outcome

- Illustrates the characteristics of normal distribution

Q.1 Determine the area under the standard normal curve (area of shaded region).

Time: 3 mnts, Score: 1

![Area = ?](image)

Scoring Indicators

0.5 + 0.3907 = 0.8907 ................................................................. (1)

Learning outcome

- Illustrates the characteristics of normal distribution.

Q.2 Indicate which of the statements below does not correctly apply to normal probability distributions:

a. they are all unimodal (ie: have a single mode)

b. they are all symmetrical

c. they all have the same mean and standard deviation

d. the area under the probability curve is always equal to 1

e. for the standard normal distribution \( \mu = 0 \) and \( \sigma = 1 \)

Time: 2 mnts, Score: 1

Scoring Indicators

Answer: c ................................................................. (1)
Learning outcome

- Illustrates the characteristics of normal distribution.

Q.3 Which of the following is not a characteristic of the normal distribution?
   a. it is a symmetrical distribution
   b. the mean is always zero
   c. the mean, median and mode are equal
   d. the total area under the curve equals one

Time: 2mins, Score: 1

Scoring Indicators

Answer: b ................................................................. (1)

Learning outcome

- Uses normal distribution in various situations.

Q.4 Let \( z \) be a standard normal value that is unknown but identifiable by position and area. If the symmetrical area between negative \( z \) and positive \( z \) is 0.9544 then the value of \( z \) must be:
   a. 2.00   b. 0.11   c. 2.50   d. 0.06

Time: 2mins, Score: 1

Scoring Indicators

Answer: a ................................................................. (1)

Learning outcome

- Uses normal distribution in various situations.

Q.5 The random variable \( X \) is normally distributed with a mean of 70 and a standard deviation of 10. Find the probability that \( X \) is within one standard deviation of the mean?
   a. 0.683   b. 0.954   c. 0.271   d. 0.340

Time: 2mins, Score: 1

Scoring Indicators

Answer: a ................................................................. (1)
Learning outcome

- Uses normal distribution in various situations.

Q.6 Probability density function of normal variable X is given by

\[ f(x) = c e^{-\frac{(x-30)^2}{2\sigma^2}}, \quad -\infty < X < \infty \]

Where c is a constant.

Find

(a) Mean of the distribution
(b) Median of the distribution
(c) \( P(X < 30) \)

Time: 10mnts, Score: 5

⚠️ Scoring Indicators

(a) mean=30 ...............................................................(1)
(b) median=30 ...........................................................(1)
(c) Standard deviation = 5 ........................................(1)
   Z score ...............................................................(1)
   Finding Table value ............................................(1)

Learning outcome

- Identifies the importance of normal distribution.

Q.7 (a) If the area between 0 and a positive value of Z (Z has a standard normal distribution) is 0.4591 then the value of Z is:
   a. 1.74  b. -1.74  c. 0.18  d. -0.18

(b) The diameters of oranges found in the orchard of an orange farm follow a normal distribution with a mean of 120mm and a standard deviation of 10mm. Find the probability of an orange in the orchard have a diameter between 110mm and 140mm?

Time: 8 mnts, Score: 1+3

⚠️ Scoring Indicators

(a) 1.74 .................................................................(1)
(b) Z score .............................................................(1)
   Finding Table value ..............................................(1)
   \( P(110 < X < 140) = 0.8186 \) ..................................(1)
Learning outcome

- Identifies the importance of normal distribution.

Q.8 (a) A larger standard deviation for a normal distribution with an unchanged mean indicates that the distribution becomes:
   a. narrower and more peaked
   b. flatter and wider
   c. more skewed to the right
   d. a change in the standard deviation does not change the shape of the distribution

(b) Student marks for a first-year Statistics class test follow a normal distribution with a mean of 63% and a standard deviation of 7%. What is the probability that a randomly selected student who wrote the test got more than 65%?

Time: 7 mnts, Score: 1+3

Scoring Indicators

(a) flatter and wider ................................................................. (1)
(b) Z score .............................................................................. (1)
    Finding Table value .............................................................. (1)
    P(X>65)=0.388 ..................................................................... (1)

Learning outcome

- Uses normal distribution in various situations.

Q.9 The distribution of monthly income of 500 workers assumed to be normal with mean Rs.2000 and standard deviation of Rs. 200. Estimate the number of workers with incomes between Rs.1800 and Rs. 2300.

Time: 8 mnts, Score: 4

Scoring Indicators

Z score .................................................................................... (1)
Finding Table value ................................................................. (1)
P(1800<X<2300) ................................................................. (1)
No of workers ........................................................................ (1)

Learning outcome

- Uses normal distribution in various situations.

Q.10 Let X be a continuous random variable that has a normal distribution with a mean of 80 and a standard deviation of 12. Find the area under the normal distribution curve
(a) from $X = 70$ to $X = 135$  
(b) to the left of $X = 27$

Time: 7mnts, Score: 4

Scoring Indicators

(a) $Z$ score ................................................................. (1)  
Area ...................................................... (1)
(b) $Z$ score ................................................................. (1)  
Area ...................................................... (1)

Learning outcome

• Illustrates the characteristics of normal distribution.

Q.11 (a) For a normal distribution coefficient of kurtosis, $\beta_2$ is ..........  
(b) If $P[Z < Z_1] = 0.89$ then find the value of $Z_1$

Time: 5mnts, Score: 3

Scoring Indicators

(a) $\beta_2 = 3$ ................................................................. (1)  
(b) application of Area property ................................. (1)  
Value of $Z_1$ ...................................................... (1)

Learning outcome

• Uses normal distribution in various situations.

Q.12 (a) Determine the total area of shaded region under the standard normal curve given below.

(b) If $X \sim N(\mu, 25)$ and $p(X > 12) = 0.1112$. What is the value of $\mu$?

Time: 6mnts, Score: 1+3

Scoring Indicators

(a) Application of Area property ................................................................. ($\frac{1}{2}$)  
Total area ...................................................... ($\frac{1}{2}$)
(b) $Z$ score ................................................................. (1)  
Application of Area property ................................................................. (1)  
Value of $\mu = 5.90$ ...................................................... (1)
Learning outcome

- Illustrates the characteristics of normal distribution.

Q.13 Determine the value of ‘a’ from the following standard normal curve.

\[
\text{Area} = 0.2256
\]

Time: 2mnts, Score: 1

**Scoring Indicators**

Area property ......................................................... \((\frac{1}{2})\)
Value of ‘a’ .............................................................. \((\frac{1}{2})\)

Learning outcome

- Uses normal distribution in various situations.

Q.14 (a) Find the value of ‘A’ if \(P(-A<Z<A)=0.8740\)

(b) The mean and standard deviation of 120 workers in a factory were 1200 and 10.33 respectively. Find the percentage of workers getting wages between Rs 900 and 1700 in the whole factory assuming that the wages are normally distributed.

Time: 7mnts, Score: 1+4

**Scoring Indicators**

(a) Area property ......................................................... \((\frac{1}{2})\)
Value of A .............................................................. \((\frac{1}{2})\)
(b) Z score .............................................................. \((1)\)
Finding Table value .................................................. \((1)\)
\(P(900<X<1700)\) .................................................. \((1)\)
Percentage of workers .............................................. \((1)\)

Learning outcome

- Illustrates the characteristics of normal distribution.

Q.15 (a) Find the value of ‘A’ if \(P(0<Z<A)=0.4838\)

(b) Probability density function of normal variable \(X\) is given by

\[
f(x) = C e^{-\frac{(x-\mu)^2}{2\sigma^2}}, -\infty < X < \infty
\]

(i) find mean and variance of \(x\) \hspace{1cm} (ii) Find the value of ‘C’

Time: 8mnts, Score: 1+3

**Scoring Indicators**

(a) Value of A ......................................................... \((1)\)
(b) (i) Mean ......................................................... \((1)\)
(ii) Variance ......................................................... \((1)\)
(iii) Value of ‘C’ ..................................................... \((1)\)
Learning outcome

- Understands the use and application of different types of sampling distributions

Q.1 The ages of badminton players of a school are 18, 15, 16, 19

(a) Find the average age of a player.

(b) If a team of 2 may be selected according to SRSWOR then find the variance of the sample means.

Time: 8 mnts, Score: 1+3

Scoring Indicators

(a) Average \( \bar{x} \) ................................................................. (1)

(b) \( V(\text{sample mean}) \) formula (SRSWOR) ........................................ (½)

Find population variance ................................................................. (2)

Substitute values in formula and finding \( V(\text{sample mean}) \) .................. (1/2)

Learning outcome

- Develops the relationship between chi-square, student’s t and F statistics

Q.2 Match the following

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ( \frac{X - \mu}{\sigma/\sqrt{n}} )</td>
<td>a Snedecor’s F variable</td>
</tr>
<tr>
<td>2 ( Z^2 )</td>
<td>b Standard normal</td>
</tr>
<tr>
<td>3 ( t^2 )</td>
<td>c Chi-square variable</td>
</tr>
<tr>
<td>4 ( \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} )</td>
<td>d t- variable</td>
</tr>
</tbody>
</table>

Time: 4 mnts, Score: 2

Scoring Indicators

1—— d ................................................................. (½)

2—— c ................................................................. (½)

3—— a ................................................................. (½)

4—— b ................................................................. (½)
Learning outcomes

- Understands the uses and application of different types of sampling distributions.

Q.3 Match the following

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} )</td>
<td>( \chi^2 )</td>
<td>Mean=0, variance = ( \frac{n}{n-2} )</td>
</tr>
<tr>
<td>( \frac{\bar{X} - \mu}{s / \sqrt{n}} )</td>
<td>t</td>
<td>Mean=0, variance = 1</td>
</tr>
<tr>
<td>( \frac{\bar{X} - \mu}{s / \sqrt{n-1}} )</td>
<td>Z</td>
<td>Mean= n, variance = 2n</td>
</tr>
</tbody>
</table>

Time: 4 mnts, Score: 3

Scoring Indicators

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} )</td>
<td>Z</td>
<td>Mean=0, variance = 1</td>
</tr>
<tr>
<td>( \frac{\bar{X} - \mu}{s / \sqrt{n-1}} )</td>
<td>t</td>
<td>Mean=0, variance = ( \frac{n}{n-2} )</td>
</tr>
<tr>
<td>( \frac{\bar{X} - \mu}{s / \sqrt{n-1}} )</td>
<td>( \chi^2 )</td>
<td>Mean= n, variance = 2n</td>
</tr>
</tbody>
</table>

1 score each

Q.4 (a) IF X is a random variable follows Chi-square distribution with 10 df then mean of X is…….

(b) A simple random sample of size 2 is drawn with replacement from a population of 4 values 50,60,64,70. Find E(\( \bar{X} \))

Time: 4 mnts, Score: 1+2

Scoring Indicators

(a) mean=10 ............................................................................................................. (1)
(b) \[ \text{E}(\bar{X}) = \mu = \frac{\sum x_i}{n} = \frac{244}{4} = 61 \] ................................................................. (1)

OR

<table>
<thead>
<tr>
<th>No of samples</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>samples</td>
<td>(50,60)</td>
<td>(50,64)</td>
<td>(50,70)</td>
<td>(60,64)</td>
<td>(60,70)</td>
<td>(64,70)</td>
<td></td>
</tr>
<tr>
<td>Sample mean</td>
<td>55</td>
<td>57</td>
<td>60</td>
<td>62</td>
<td>65</td>
<td>67</td>
<td>366</td>
</tr>
</tbody>
</table>

Samples ............................................................................................................. (1)
Sample mean .................................................................................................... (1/2)
\[ \text{E}(\bar{X})=366/6=61 \] ........................................................................... (1/2)
Learning outcomes

- Understands the uses and application of different types of sampling distributions.

Q5. Fill up the missing values for the table given below:

<table>
<thead>
<tr>
<th>Sl No</th>
<th>Value of Variate</th>
<th>( \alpha )</th>
<th>d.f.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \chi^2 = 22.36 )</td>
<td>0.05</td>
<td>—</td>
</tr>
<tr>
<td>2</td>
<td>( t = 2.47 )</td>
<td>—</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>( F = )</td>
<td>0.01</td>
<td>(8,15)</td>
</tr>
</tbody>
</table>

Time: 3 mnts, Score: 3

Scoring Indicators

<table>
<thead>
<tr>
<th>Sl No</th>
<th>Value of Variate</th>
<th>( \alpha )</th>
<th>d.f.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \chi^2 = 22.36 )</td>
<td>0.05</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>( t = 2.47 )</td>
<td>0.025</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>( F = 3.89 )</td>
<td>0.01</td>
<td>(8,15)</td>
</tr>
</tbody>
</table>

Learning outcomes

- Understands the uses and application of different types of sampling distributions.

Q6. Fill up the missing values for the table given below:

<table>
<thead>
<tr>
<th>Value of variate</th>
<th>( \alpha )</th>
<th>d.f.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X^2 = 22.36 )</td>
<td>—</td>
<td>13</td>
</tr>
<tr>
<td>( t = 2.779 )</td>
<td>0.005</td>
<td>—</td>
</tr>
</tbody>
</table>

Time: 2 mnts, Score: 2

Scoring Indicators

<table>
<thead>
<tr>
<th>Value of variate</th>
<th>( \alpha )</th>
<th>d.f.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X^2 = 22.36 )</td>
<td>0.05</td>
<td>13</td>
</tr>
<tr>
<td>( t = 2.779 )</td>
<td>0.005</td>
<td>26</td>
</tr>
</tbody>
</table>

I score each

Learning outcomes

- Understands the uses and application of different types of sampling distributions.

Q7. The ratio of two Chi-square variable is \( \) variable.

(a) \( Z \)  \( \) (b) \( t \)  \( \) (c) Chi-Square  \( \) (d) \( F \)

Time: 2 mnts, Score: 1

Scoring Indicators

\( F \) variable ................................................................. (1)
Learning outcomes

- Differentiates between parameter and statistics and to exemplify them.

Q.8. In a statistical investigation a sample of size 22 has a mean of 93.1 and standard deviation 7. The sample is drawn from a population with size 4052 has a mean of 94.3 and standard deviation 7.6. List out the values of the parameters and statistics involved in this study.

Time: 3 mnts, Score: 2

Scoring Indicators

Values of the parameters ................................................................. (1)
Values of the statistics ................................................................. (1)

-------------------

Learning outcomes

- Understands the uses and application of different types of sampling distributions.

Q.9. The \( \chi^2 \) random variable has degrees of freedom 10. Then what would be the values of \( k \), if \( P(\chi^2 > k) = 0.01 \).

Time: 2 mnts, Score: 1

Scoring Indicators

Value of \( k \) ......................................................................................... (1)

-------------------

Learning outcomes

- Understands the uses and application of different types of sampling distributions.

Q.10. A random variable ‘t’ has \( t \) distribution with 8 degrees of freedom. Obtain the value of \( a \) such that \( P(t > a) = 0.01 \)

Time: 2 mnts, Score: 1

Scoring Indicators

Value of \( a \) ......................................................................................... (1)
ESTIMATION OF PARAMETERS

Learning Outcome
• Lists out the desirable properties of a good estimator

Q.1 Which of the following is not a desirable property for a good estimator?
   a) Sufficiency  b) Efficiency  c) Randomness  d) Unbiasedness

Score : 1, Time : 1 Minute

⚠ Scoring Indicators
Randomness ................................................................. (1)

Learning Outcome
• Examine the properties of estimators

Q.2 An estimator ‘t’ is said to be an unbiased estimator for the parameter θ if
   b) E(t) = 0  b) E(t) = θ  c) V(t) = 0  d) V(t) = θ

Score : 1, Time : 1 Minute

⚠ Scoring Indicators
E(t) = θ ................................................................. (1)

Learning Outcome
• Describes point estimates and interval estimates

Q.3 The population mean is expected in between 122 and 128. This statement is an example for .................
   a) Interval Estimation
   b) Point Estimation
   c) Both Interval and Point Estimation
   d) Neither interval nor point estimation

Score : 1, Time : 1 Minutes

⚠ Scoring Indicators
a) Interval Estimation .................................................. (1)
Learning Outcome

- Recognises the estimator and estimate

Q4 In an inferential statistics, the sample mean obtained is 56. Then 56 is the ..............
   a) Estimator of the population mean
   b) Estimator of the sample mean
   c) Estimate of the sample mean
   d) Estimate of the population mean

Score: 1 Score, Time: 1 Minutes

Scoring Indicators

d) Estimate of the population mean ......................................................... (1)

Learning Outcome

- Examines the properties of estimators

Q5 Choose the correct answer
   a) Sample mean is a Sufficient estimator but not Unbiased.
   b) Sample mean is a Consistent estimator but not Unbiased.
   c) Sample mean is not a Sufficient estimator but Unbiased.
   d) Sample mean is a Consistent estimator and Unbiased.

Score: 1, Time: 1 Minute

Scoring Indicators

d) Sample mean is a Consistent estimator and Unbiased. ......................... (1)

Learning Outcome

- Examines the properties of estimators

Q6 An estimator $t$ is consistent for $\theta$, if
   a) $E(t) = \theta$ and $V(t) = 0$
   b) $E(t) \rightarrow \theta$ and $V(t) = 0$
   c) $E(t) = \theta$ and $V(t) \rightarrow 0$
   d) $E(t) \rightarrow \theta$ and $V(t) \rightarrow 0$

Score: 1 Score, Time: 2 Minutes

Scoring Indicators

d) $E(t) \rightarrow \theta$ and $V(t) \rightarrow 0$ .................................................. (1)
Learning Outcome

- Examines the properties of estimators

Q.7 An estimator $t_1$ is said to be efficient than the estimator $t_2$, if

a) $V(t_1) < V(t_2)$

b) $V(t_1) > V(t_2)$

c) $V(t_1) \leq V(t_2)$

d) $V(t_1) \geq V(t_2)$

Score: 1 Score, Time: 1 Minutes

⚠ Scoring Indicators

a) $V(t_1) < V(t_2)$ ................................................................. (1)

Learning Outcome

- Examines the properties of estimators

Q.8 $X_1, X_2, X_3$ and $X_4$ are sample values drawn from a normal population with mean $\mu$ and standard deviation $\sigma$. $3X_1 + 4X_2 + kX_3$ is an unbiased estimator for the population mean. Then $k =$ .......

a) 0       b) 1       c) 2       d) 3

Score: 1, Time: 2 Minute

⚠ Scoring Indicators

c) 2 ................................................................. (1)

Learning Outcome

- Examines the properties of estimators

Q.9 $X_1, X_2, X_3, X_4$ is a sample drawn from a normal population with mean $\mu$ and standard deviation $\sigma$. Show that $T = 4X_1 + X_2 - 3X_3 - X_4$ is an unbiased estimator for the population mean.

Score: 2, Time: 3 Minutes

⚠ Scoring Indicators

$$E(X_1) = E(X_2) = E(X_3) = E(X_4) = \mu .................................................. (\frac{1}{2})$$

$$E(T) = 4E(X_1) + E(X_2) - 3E(X_3) - E(X_4) ................................. (\frac{1}{2})$$

$$= 4\mu + \mu - 3\mu = \mu .................................................. (\frac{1}{2})$$

$\therefore$ $T$ is unbiased. .................................................. (\frac{1}{2})
Learning Outcome

- Examines the properties of estimators

Q.10 $X_1, X_2$ and $X_3$ is a sample drawn from a normal population with mean $\mu$ and standard deviation $\sigma$. Check the unbiasedness of the estimators

$$T = X_1 + X_2 - X_3$$
$$U = 3X_1 + X_2 - 2X_3$$
$$V = \frac{2X_1 + X_2 + X_3}{4}$$

Score: 3, Time: 6 Minutes

Scoring Indicators

- $E(T) = \mu$, $\therefore T$ is unbiased. ................................................................. (1)
- $E(U) = 2\mu$, $\therefore U$ is not unbiased ................................................................. (1)
- $E(V) = \mu$, $\therefore V$ is unbiased................................................................. (1)

Learning Outcome

- Examines the properties of estimators

Q.11 $X_1, X_2$ and $X_3$ is a sample drawn from a normal population with mean $\mu$ and standard deviation $\sigma$. Find the efficient estimator among the following.

$$T_1 = 2X_1 + X_2 - X_3$$
$$T_2 = 2X_1 + 2X_2 - 2X_3$$
$$T_3 = X_1 + X_2 - X_3$$

Score: 5, Time: 8 Minutes

Scoring Indicators

- Test for unbiasedness .................................................................................... (2)
- Determination of Variance ........................................................................... (2)
- Efficiency ........................................................................................................ (1)

Learning Outcome

- Estimates the parameter by using method of moments

Q.12 Estimate the population mean by using the method of moments from the following sample.

51, 53, 57, 54, 55, 59 and 55

Score: 3, Time: 5 Minutes
Scoring Indicators

Sample moment ................................................................. (1)
Equation ........................................................................... (1)
Conclusion ......................................................................... (1)

Learning Outcome :

• Constructs confidence interval for population mean.

Q.13 A random sample of size 120 is selected from population of iron blocks. The mean and standard deviation of the sample are 4012g and 68g respectively. Find a 95 % confidence interval for the population mean.

Score : 4, Time : 7 Minutes

Scoring Indicators

Equation ........................................................................... (1)
Process ............................................................................. (2)
Result ............................................................................... (1)

Learning Outcome

• Recognises the estimator and estimate

Q.14 Estimator and estimate are closely related in estimation of parameters. How?

Score : 3, Time : 5 Minutes

Scoring Indicators

Estimator ............................................................................ (1)
Estimate ............................................................................. (1)
Relation ............................................................................. (1)

Learning Outcome

• Describes point estimates and interval estimates

Q.15 Distinguish between point estimation and interval estimation.

Score : 3, Time : 5 Minutes

Scoring Indicators

Point Estimation ................................................................. (1)
Interval estimation .......................................................... (1)
Conclusion ......................................................................... (1)
Learning Outcome

- Lists out the desirable properties of a good estimator

Q.16 A good estimator is expected to have some properties. Explain.

Score: 4, Time: 7 Minutes

Score Indicators

- Unbiasedness ................................................................. (1)
- Consistency ................................................................. (1)
- Efficiency ................................................................. (1)
- Sufficiency ................................................................. (1)

Learning Outcome

- Lists out the desirable properties of a good estimator.

Q.17 An unbiased estimator may not be consistent. Justify.

Score: 3, Time: 6 Minutes

Score Indicators

- Unbiasedness ................................................................. (1)
- Consistancy ................................................................. (1)
- Justification ................................................................. (1)
Learning Outcome

- Lists out the desirable properties of a good estimator

Q.1 Which of the following is not a desirable property for a good estimator?
   a) Sufficiency    b) Efficiency    c) Randomness    d) Unbiasedness

Score: 1, Time: 1 Minute

⚠️ Scoring Indicators

Randomness ........................................................................................................... (1)

Learning Outcome

- Examine the properties of estimators

Q.2 An estimator ‘t’ is said to be an unbiased estimator for the parameter \( \theta \) if
   b) \( E(t) = 0 \)    b) \( E(t) = \theta \)    c) \( V(t) = 0 \)    d) \( V(t) = \theta \)

Score: 1, Time: 1 Minute

⚠️ Scoring Indicators

\( E(t) = \theta \) ........................................................................................................... (1)

Learning Outcome

- Describes point estimates and interval estimates

Q.3 The population mean is expected in between 122 and 128. This statement is an example for .................
   a) Interval Estimation
   b) Point Estimation
   c) Both Interval and Point Estimation
   d) Neither interval nor point estimation

Score: 1, Time: 1 Minutes

⚠️ Scoring Indicators

a) Interval Estimation ........................................................................................................... (1)
Learning Outcome

- Recognises the estimator and estimate

Q.4 In an inferential statistics, the sample mean obtained is 56. Then 56 is the ............
   a) Estimator of the population mean
   b) Estimator of the sample mean
   c) Estimate of the sample mean
   d) Estimate of the population mean

Score: 1 Score, Time: 1 Minutes

Scoring Indicators

d) Estimate of the population mean .......................................................... (1)

Learning Outcome

- Examines the properties of estimators

Q.5 Choose the correct answer
   a) Sample mean is a Sufficient estimator but not Unbiased.
   b) Sample mean is a Consistent estimator but not Unbiased.
   c) Sample mean is not a Sufficient estimator but Unbiased.
   d) Sample mean is a Consistent estimator and Unbiased.

Score: 1, Time: 1 Minute

Scoring Indicators

d) Sample mean is a Consistent estimator and Unbiased. ......................... (1)

Learning Outcome

- Examines the properties of estimators

Q.6 An estimator $t$ is consistent for $\theta$, if
   a) $E(t) = \theta$ and $V(t) = 0$
   b) $E(t) \to \theta$ and $V(t) = 0$
   c) $E(t) = \theta$ and $V(t) \to 0$
   d) $E(t) \to \theta$ and $V(t) \to 0$

Score: 1 Score, Time: 2 Minutes

Scoring Indicators

d) $E(t) \to \theta$ and $V(t) \to 0$ ........................................................................ (1)
Learning Outcome

- Examines the properties of estimators

Q.7 An estimator $t_1$ is said to be efficient than the estimator $t_2$, if.............

a) $V(t_1) < V(t_2)$
b) $V(t_1) > V(t_2)$
c) $V(t_1) \leq V(t_2)$
d) $V(t_1) \geq V(t_2)$

Score: 1 Score, Time: 1 Minutes

⚠ Scoring Indicators

a) $V(t_1) < V(t_2)$ ................................................................. (1)

Learning Outcome

- Examines the properties of estimators

Q.8 $X_1$, $X_2$, and $X_3$ are sample values drawn from a normal population with mean $\mu$ and standard deviation $\sigma$. $3X_1 - 4X_2 + kX_3$ is an unbiased estimator for the population mean. Then $k =$ ........

a) 0  b) 1  c) 2  d) 3

Score: 1, Time: 2 Minute

⚠ Scoring Indicators

c) 2 ................................................................. (1)

Learning Outcome

- Examines the properties of estimators

Q.9 $X_1$, $X_2$, $X_3$, and $X_4$ is a sample drawn from a normal population with mean $\mu$ and standard deviation $\sigma$. Show that $T = 4X_1 + X_2 - 3X_3 - X_4$ is an unbiased estimator for the population mean.

Score: 2, Time: 3 Minutes

⚠ Scoring Indicators

$E(X_1) = E(X_2) = E(X_3) = E(X_4) = \mu$ ........................................ (½)

$E(T) = 4E(X_1) + E(X_2) - 3E(X_3) - E(X_4)$ ................................ (½)

$= 4\mu + \mu - 3\mu - \mu = \mu$ .................................................. (½)

$\therefore T$ is unbiased. .................................................. (½)
Learning Outcome

- Examines the properties of estimators

Q.10  \( X_1, X_2 \text{ and } X_3 \) is a sample drawn from a normal population with mean \( \mu \) and standard deviation \( \sigma \). Check the unbiasedness of the estimators

\[
T = X_1 + X_2 - X_3 \\
U = 3X_1 + X_2 - 2X_3 \\
V = \frac{2X_1 + X_2 + X_3}{4}
\]

Score: 3, Time: 6 Minutes

\[\text{Scoring Indicators}\]

\[
E(T) = \mu, \quad \therefore \text{T is unbiased.} \quad (1)
\]

\[
E(U) = 2\mu, \quad \therefore \text{U is not unbiased} \quad (1)
\]

\[
E(V) = \mu, \quad \therefore \text{V is unbiased.} \quad (1)
\]

Learning Outcome

- Examines the properties of estimators

Q.11  \( X_1, X_2 \text{ and } X_3 \) is a sample drawn from a normal population with mean \( \mu \) and standard deviation \( \sigma \). Find the efficient estimator among the following.

\[
T_1 = 2X_1 + X_2 - X_3 \\
T_2 = 2X_1 + 2X_2 - 2X_3 \\
T_3 = X_1 + X_2 - X_3
\]

Score: 5, Time: 8 Minutes

\[\text{Scoring Indicators}\]

- Test for unbiasedness \( \quad (2) \)
- Determination of Variance \( \quad (2) \)
- Efficiency \( \quad (1) \)

Learning Outcome

- Estimates the parameter by using method of moments

Q.12  Estimate the population mean by using the method of moments from the following sample.

\[
51, 53, 57, 54, 55, 59 \text{ and } 55
\]

Score: 3, Time: 5 Minutes
Scoring Indicators

Sample moment ......................................................... (1)
Equation ................................................................. (1)
Conclusion .............................................................. (1)

Learning Outcome :

• Constructs confidence interval for population mean.

Q.13 A random sample of size 120 is selected from population of iron blocks. The mean and standard deviation of the sample are 4012g and 68g respectively. Find a 95 % confidence interval for the population mean.

Score : 4, Time : 7 Minutes

Scoring Indicators

Equation ................................................................. (1)
Process ..................................................................... (2)
Result ....................................................................... (1)

Learning Outcome

• Recognises the estimator and estimate

Q.14 Estimator and estimate are closely related in estimation of parameters. How?

Score : 3, Time : 5 Minutes

Scoring Indicators

Estimator ................................................................. (1)
Estimate ................................................................. (1)
Relation ................................................................. (1)

Learning Outcome

• Describes point estimates and interval estimates

Q.15 Distinguish between point estimation and interval estimation.

Score : 3, Time : 5 Minutes

Scoring Indicators

Point Estimation ......................................................... (1)
Interval estimation .................................................... (1)
Conclusion ............................................................ (1)
Learning Outcome

- Lists out the desirable properties of a good estimator.

Q.16 A good estimator is expected to have some properties. Explain.

Score: 4, Time: 7 Minutes

 대하여 Scoring Indicators

Unbiasedness ................................................................. (1)
Consistency ................................................................. (1)
Efficiency ................................................................. (1)
Sufficiency ................................................................. (1)

Learning Outcome

- Lists out the desirable properties of a good estimator.

Q.17 An unbiased estimator may not be consistent. Justify.

Score: 3, Time: 6 Minutes

 대하여 Scoring Indicators

Unbiasedness ................................................................. (1)
Consistancy ................................................................. (1)
Justification ................................................................. (1)
Learning Outcome

- Explains the concept of ANOVA

Q.1 Which of the following statistical test is suitable for testing the significance of difference of several means?

a) t - Test  b) F - Test  c) Z - Test  d) Chi-Square Test

Score : 1, Time : 2 Minutes

⚠️ Scoring Indicators

b) F - Test ................................................................. (1)

Learning Outcome

- Explains the concept of ANOVA

Q.2 Analysis of Variance is a statistical tool to test ......................

a) Mean  b) Variance  c) Uniformity  d) Homogeneity

Score : 1, Time : 2 Minutes

⚠️ Scoring Indicators

a) Mean............................................................................. 1

Learning Outcome

- Explains the concept of ANOVA

Q.3 Which of the following is the null hypothesis for ANOVA?

a) $\mu_1 = \mu_2 = \mu_3 = \ldots = \mu_k$

b) $\mu_1 > \mu_2 > \mu_3 > \ldots > \mu_k$

c) $\mu_1 < \mu_2 < \mu_3 < \ldots < \mu_k$

d) $\mu_1 \neq \mu_2 \neq \mu_3 \neq \ldots \neq \mu_k$

Score : 1, Time : 2 Minutes

⚠️ Scoring Indicators

a) $\mu_1 = \mu_2 = \mu_3 = \ldots = \mu_k$ .......................................... (1)
Learning Outcome

- Explains the concept of ANOVA

Q.4 Which of the following assumption is not related to ANOVA?
   a) Additivity b) Homogeneity
   c) Normality d) Multiplicativity

Score: 1, Time: 2 Minutes

Scoring Indicators

d) Multiplicativity

Learning Outcome

- Constructs ANOVA Table.

Q.5 Five treatments are applied in 22 units and conducted an ANOVA. Then the error degrees of freedom is .......... 
   a) 4 b) 21 c) 17 d) 18

Score: 1, Time: 2 Minutes

Scoring Indicators

c) 17

Learning Outcome

- Identifies the causes of variations.

Q.6 Treatment variations in the ANOVA is due to .........................
   a) Assignable Cause b) Chance Cause
   c) Either Assignable Cause Or Chance Cause d) Neither Assignable Cause nor Chance Cause

Score: 1, Time: 2 Minutes

Scoring Indicators

a) Assignable Cause

Learning Outcome

- Explains the concept of ANOVA

Q.7 ANOVA is developed by ................
   a) W Z Gozzet b) Karl Pearson
   c) P C Mahalanobis d) R A Fisher

Score: 1, Time: 2 Minutes
Scoring Indicators

d) R A Fisher

Learning Outcome

- Interprets ANOVA Table.

Q.8 In Analysis of Variance, the null hypothesis is rejected when

a) \( F > F_{\alpha} \)  

b) \( F < F_{\alpha} \)  

c) \( F \leq F_{\alpha} \)  

d) \( F \geq F_{\alpha} \)

Score: 1, Time: 2 Minutes

Scoring Indicators

d) \( F \geq F_{\alpha} \)

Learning Outcome

- Interprets ANOVA Table.

Q.9 An ANOVA is conducted to study the significance of difference of efficiencies of three dot matrix printers by taking samples of sizes 4, 5 and 3 respectively. The \( F \) value obtained is 7.29. What is your conclusion on the efficiency of the printers at 5% level of significance?

Score: 2, Time: 4 Minutes

Scoring Indicators

Find \( F_{0.05} \)

Reject \( H_0 \) if \( F \geq F_{0.05} \)

Learning Outcome

- Interprets ANOVA Table.

Q.10 In an ANOVA the mean sum of squares due to treatments and the mean sum of squares due to error are 22.35 and 3.92 with 4 and 20 degrees of freedom respectively. Write the conclusion of the test.

Score: 3, Time: 5 Minutes

Scoring Indicators

Preparation of ANOVA Table

Conclusion
Learning Outcome

- Constructs ANOVA Table, Interprets ANOVA table

Q.11  In an ANOVA the sum of squares between samples and the sum of squares within sample are 80 and 96 with 5 and 18 degrees of freedom respectively. Draw the ANOVA table and write your inference.

Score : 4, Time : 7 Minutes

 предостига пример

<table>
<thead>
<tr>
<th>Scoring Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preparation of ANOVA Table ................................................................. (3)</td>
</tr>
<tr>
<td>Conclusion ................................................................. (1)</td>
</tr>
</tbody>
</table>

Learning Outcome

- Constructs ANOVA Table, Interprets ANOVA Table.

Q.12  In an ANOVA the total sum of squares and the sum of squares between samples are 197.2 and 165.1 respectively, when five treatment are applied at 22 different plots. Prepare the ANOVA table and test whether the treatment effects are significantly different or not at 5% level of significance.

Score : 4, Time : 8 Minutes

 предостига пример

<table>
<thead>
<tr>
<th>Scoring Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preparation of ANOVA Table ................................................................. (3)</td>
</tr>
<tr>
<td>Conclusion ................................................................. (1)</td>
</tr>
</tbody>
</table>

Learning Outcome

- Constructs ANOVA Table, Interprets ANOVA Table.

Q.13  Complete the following ANOVA Table and interpret the result.

<table>
<thead>
<tr>
<th>Source</th>
<th>Df</th>
<th>Sum of Squares</th>
<th>Mean Sum of Squares</th>
<th>F</th>
<th>F₀₀₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Samples</td>
<td>-</td>
<td>67</td>
<td>-</td>
<td>3.1</td>
<td>-</td>
</tr>
<tr>
<td>Within Samples</td>
<td>11</td>
<td>-</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>14</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Score : 4, Time : 8 Minutes

 предостига пример

<table>
<thead>
<tr>
<th>Scoring Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSW and TSS ................................................................. (1)</td>
</tr>
<tr>
<td>MSB and MSW ................................................................. (1)</td>
</tr>
<tr>
<td>df and F₀₀₅ ................................................................. (1)</td>
</tr>
<tr>
<td>Conclusion ................................................................. (1)</td>
</tr>
</tbody>
</table>
Learning Outcome

- Constructs ANOVA Table and 10.5 Interprets ANOVA Table.

Q.14  The yields from four varieties of plants are given below. Analyse the data and test whether the yields from various varieties of plants are significantly different or not at 5% level of significance.

<table>
<thead>
<tr>
<th>Variety</th>
<th>Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>12 14 14 13 15</td>
</tr>
<tr>
<td>B</td>
<td>10 12 11 13</td>
</tr>
<tr>
<td>C</td>
<td>18 18 20 19 19</td>
</tr>
<tr>
<td>D</td>
<td>11 12 12 14</td>
</tr>
</tbody>
</table>

Score : 5, Time : 10 Minutes

⚠️ Scoring Indicators

- CF, TSS – ................................................................. (1)
- SSB, SSW and df – .................................................. (1)
- Anova Table .............................................................. (2)
- Conclusion .............................................................. (1)

Learning Outcome

- Identifies the causes of variations.

Q.15  Variation within samples and variation between samples are very important in ANOVA. Substantiate.

Score : 3, Time : 5 Minutes

⚠️ Scoring Indicators

- Variation within samples – .................................................. (1)
- Variation between samples – .................................................. (1)
- Their significance in Anova – .................................................. (1)
STATISTICAL QUALITY CONTROL

Learning Outcomes

- Explains the concept of control charts for variables and attributes
- Constructs control charts for variables and attributes

Q.1

(a) Average weight of the contents of a box of apples can be monitored by ............ chart
   (i) C-chart     (ii) p-chart     (iii) x-bar     (iv) R-chart

(b) Range of 15 random samples of size 7 were 20, 34, 10, 18, 20, 16, 21, 9, 11, 14, 22, 19, 22, 20 and 11. Construct a control chart for range and determine whether the process is in control or not.

Score :4, Time: 10 mnts

Scoring Indicators

a) 
   (iii) 

b) \( K = 15 \), \( D_3 = 0.076 \), \( D_4 = 1.924 \), \( n = 7 \)
   \( \sum R = 267 \), \( \frac{R}{R} = 267/15 = 17.8 \) .................................................. (1)
   \( \text{UCL} = D_4 \frac{\overline{R}}{R} = 34.25 \) .................................................. (½)
   \( \text{LCL} = D_3 \frac{\overline{R}}{R} = 1.35 \) .................................................. (¼)
   \( \overline{R} = 17.8 \).
   Construction ................................................................. (2)

Learning Outcomes

- Identifies the definition of quality, SQC, and statistical process
- Constructs control charts for variables and attributes

Q.2

a) process control is carried out:
   i) before production     ii) after production
   iii) during production     iv) all the above

b) Ten samples of 5 items each are drawn from the output of a process. Range and SD of the Samples are computed. Total of \( R \) for ten samples is 210. Determine the control limits.

Score : 3, Time: 5 mnts
Scoring Indicators

a) (iii) .................................................................................................................. (1)

b) \( n=5, \ k=10 \) , Total of \( R = 210 \) ........................................................................... \( \frac{1}{2} \)
\[
CL = \overline{R} = \frac{\sum R}{10} = 21
\]
\( UCL = D_4 \overline{R} = 2.115 \times 21 = 44.415 \) as \( D_4 = 2.115 \) ......................... \( \frac{1}{2} \)
\( LCL = D_3 \overline{R} = 0 \) as \( D_3 = 0 \) ................................................................. \( \frac{1}{2} \)

Learning Outcomes

- Identifies the definition of quality, SQC, and statistical process
- Explains the concept of control charts for variables and attributes

Q.3 a) The dividing lines between random and non-random deviations from mean of the distribution are known as:

(i) Upper control limit
(ii) Lower control limit
(iii) Control limits
(iv) Two sigma limits

b) 10 samples of 3 items each were taken from the output of a process and measures of \( \overline{X} \) and \( R \) were obtained. Determine the control limits for \( \overline{X} \) and \( R \) chart.

Sample no | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10
---|---|---|---|---|---|---|---|---|---|---
\( \overline{X} \) | 128 | 127 | 129 | 149 | 152 | 131 | 144 | 120 | 112 | 134
\( \overline{R} \) | 18 | 23 | 6 | 7 | 31 | 24 | 14 | 16 | 9 | 23

Score: 5, Time: 5 mnts

Scoring Indicators

a) (iii) .................................................................................................................. (1)

b) \( \overline{X} = 132.6 = CL \) .............................................................................. \( \frac{1}{2} \)
\[
\overline{R} = 17.1
\]
\( UCL = 150.04 \) where \( UCL = \overline{X} + A_2 \overline{R} \) ........................................ \( \frac{1}{2} \)
\( LCL = 115.16 \) where \( LCL = \overline{X} - A_2 \overline{R} \) ........................................ \( \frac{1}{2} \)
Construction ................................................................................................. (2)

Learning Outcomes

- Identifies the definition of quality, SQC, and statistical process
- Distinguishes between chance causes and assignable causes
- Explains the concept of control charts for variables and attributes
- Constructs control charts for variables and attributes
Q.4 Match the following

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>i. Chart used to monitor attributes</td>
<td>Mean chart</td>
</tr>
<tr>
<td>ii. Central tendency of a process is monitored</td>
<td>P-chart</td>
</tr>
<tr>
<td>iii. Dispersion of a process is monitored</td>
<td>Range chart</td>
</tr>
<tr>
<td>iv. The constant ( D_2 ) is used</td>
<td>Z chart</td>
</tr>
</tbody>
</table>

Score: 2, Time: 2 mnts

**Scoring Indicators**

i. P-chart  
ii. Mean chart  
iii. Range chart  
iv. Range chart

---

**Learning Outcomes**

- Constructs control charts for variables and attributes

Q.5  

a) The variation in a product which can’t be controlled is called .......... Variation.

b) What are the uses of SQC?

Score: 3, Time: 5 mnts

**Scoring Indicators**

a) Chance................................................................................................................... (1)  

b) Any 4 points............................................................................................................... (2)

---

**Learning Outcomes**

- Constructs control charts for variables and attributes

Q.6 Use of the sample range to estimate variability can also be applied to a company of Potato chips operation. A quality control inspector selects 4 samples with 5 observations each, measuring the volume of chips per bag. If the average range for the samples is 0.2 ounces and the average mean of the observations is 12.5 ounces, develop three-sigma control limits for the bottling operation.

Score 4, Time: 4 mnts

**Scoring Indicators**

\[
\bar{R} = 0.2  \\
\bar{X} = 12.5 \text{ ounces}  \\
A_2 = .58  \\
UCL = \bar{X} + A_2 \bar{R} = 12.5 + (.58)(0.2) = 12.38  \\
LCL = \bar{X} - A_2 \bar{R} = 12.5 + (-.58)(2) = 12.62
\]
Learning Outcomes

- Constructs control charts for variables and attributes

Q.7 A machine is to set to deliver packets of a given weight. 10 samples of the size 5 each were recorded. Below are given relevant data

<table>
<thead>
<tr>
<th>Sample no</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>15</td>
<td>17</td>
<td>15</td>
<td>18</td>
<td>17</td>
<td>14</td>
<td>18</td>
<td>15</td>
<td>17</td>
<td>16</td>
</tr>
<tr>
<td>Range</td>
<td>7</td>
<td>7</td>
<td>4</td>
<td>9</td>
<td>8</td>
<td>7</td>
<td>12</td>
<td>4</td>
<td>11</td>
<td>5</td>
</tr>
</tbody>
</table>

Calculate control limits, Draw control chart and comment on the state control.

Score: 6, Time: 10 mnts

\[ \bar{X} = 16.2 \quad \text{and} \quad \bar{R} = 7.4 \]

UCL = \[ \bar{X} + A_2 \bar{R} = 20.492 \]

LCL = \[ \bar{X} - A_2 \bar{R} = 11.908 \]

R chart

UCL = \[ D_4 \bar{R} = 15.614 \]

LCL = \[ D_3 \bar{R} = 0 \]

Construction:

Comment:

Learning Outcomes

- Constructs control charts for variables and attributes

Q.8 The following data refer to visual defects found at inspection of the first 10 samples of size 100.

Construct control chart and comment on it.

<table>
<thead>
<tr>
<th>Sample no</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>No of defectives</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

Score: 6, Time: 12 mnts

n = 100 \[ \bar{p} = 0.02 \]

UCL formula and value = 6.2

LCL formula and value = (-2.2)

Construction and comment
Learning Outcome

- Identifies time series

Q.1 Time Series is ....................
   a) Geographically organised data
   b) Classified according to attributes
   c) Quantitative classification of data
   d) Chronologically arranged data

Score: 1 Score, Time: 2 Minutes

⚠️ Scoring Indicators
   
   d) Chronologically arranged data ......................................................... (1)

Learning Outcome

- Differentiates the components of time series

Q.2 Which of the following is periodic movement in time series?
   a) Cyclical variation only
   b) Seasonal variation only
   c) Both cyclical variation and seasonal variation
   d) Neither cyclical variation nor seasonal variation

Score: 1 Score, Time: 2 Minutes

⚠️ Scoring Indicators

   c) Cyclical variation and seasonal variation ........................................... (1)
Learning Outcome

- Differentiates the components of time series

Q.3 The sales of flowers during the onam festivals are associated to the .............. component in the time series

a) Cyclical variation  b) Seasonal variation

b) Secular trend  d) Irregular variation

Score : 1 Score, Time : 2 Minutes

Scoring Indicators

b)  Seasonal variation ................................................................. (1)

Learning Outcome

- Differentiates the components of time series

Q.4 The depression in the market due to the withdrawal of existing currencies by the government is ................. component of a time series.

a) Cyclical variation  b) Seasonal variation

c) Secular trend  d) Irregular variation

Score : 1 Score, Time : 2 Minutes

Scoring Indicators

d)  Irregular variation ................................................................. (1)

Learning Outcome

- Estimates the future values by using time series analysis

Q.5 Which of the following techniques is NOT USED in time series for the trend analysis?

a) Method of moving averages  b) Method of least squares

c) Method of semi averages  d) Method of moments

Score : 1 Score, Time : 2 Minutes

Scoring Indicators

d)  Method of moments ................................................................. (1)
Learning Outcome

- Differentiates the components of time series

Q.6 Various components may be involved in a time series. Substantiate.

Score : 4 Score, Time : 4 Minutes

Scoring Indicators

Cyclical variation ................................................................. (1)
Seasonal variation ............................................................... (1)
Secular trend ....................................................................... (1)
Irregular variation ............................................................... (1)

Learning Outcome

- Differentiates the components of time series

Q.7 Match the following

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>The hike in sales of sweets in the Deepavali.</td>
<td>Secular Trend</td>
</tr>
<tr>
<td>The hike in the sales of chicken due to the declaration of harthal in Kerala</td>
<td>Seasonal Variation</td>
</tr>
<tr>
<td>The hike in the sales of newspapers in Kerala</td>
<td>Cyclic Variation</td>
</tr>
<tr>
<td>The hike and drop of the sales of handmade and craft items.</td>
<td>Irregular Variation</td>
</tr>
</tbody>
</table>

Score : 2 Score, Time : 5 Minutes

Scoring Indicators

$4 \times \frac{1}{2} = 2$

Learning Outcome

- Estimates the future values by using time series analysis

Q.8 Fit a trend line to the following time series by using free hand curve method

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>3</td>
<td>8</td>
<td>4</td>
<td>9</td>
<td>12</td>
<td>13</td>
<td>10</td>
<td>16</td>
<td>19</td>
<td>24</td>
<td>21</td>
<td>23</td>
</tr>
</tbody>
</table>

Score : 3 Score, Time : 6 Minutes

Scoring Indicators

Ploting ............................................................................. (2)
Curve Fitting ..................................................................... (1)
Learning Outcome

- Estimates the future values by using time series analysis

Q.9 Consider the following time series

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>62</td>
<td>61</td>
<td>66</td>
<td>70</td>
<td>68</td>
<td>79</td>
<td>71</td>
<td>78</td>
<td>84</td>
<td>88</td>
<td>87</td>
</tr>
</tbody>
</table>

a) Obtain the semi averages  
b) Plot the values in the graph  
c) Draw the trend line by using semi averages

Score: 4 Score, Time: 7 Minutes

Scoring Indicators

- Semi Average ................................................................. (1)  
- Ploting ................................................................. (2)  
- Curve Fitting ................................................................. (1)

Learning Outcome

- Estimates the future values by using time series analysis

Q.10 Fit a straight line trend to the following time series by using method of least squares.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>3</td>
<td>7</td>
<td>9</td>
<td>6</td>
<td>7</td>
<td>9</td>
<td>11</td>
<td>10</td>
<td>18</td>
<td>15</td>
</tr>
</tbody>
</table>

Score: 4 Score, Time: 7 Minutes

Scoring Indicators

- Normal Equations ................................................................. (2)  
- Solution of equations ................................................................. (1)  
- Trend Equation ................................................................. (1)

Learning Outcome

- Estimates the future values by using time series analysis

Q.11 The production of a company for the last 12 years is given below.

a) Fit a straight line trend by using method of least squares.

b) Estimate the expected production of the company in the year 2019.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Production (in 000’s)</td>
<td>11</td>
<td>14</td>
<td>17</td>
<td>16</td>
<td>18</td>
<td>15</td>
<td>19</td>
<td>24</td>
<td>21</td>
<td>22</td>
<td>22</td>
<td>27</td>
</tr>
</tbody>
</table>

Score: 5 Score, Time: 10 Minutes
Scoring Indicators

Normal Equations ................................................................. (2)
Solution of equations .......................................................... (1)
Trend Equation ................................................................. (1)
Estimation ................................................................. (1)

Learning Outcome

- Estimates the future values by using time series analysis
- Interprets trend lines

Q.12 The sales of a company for the last 11 years are given below.

a) Fit a straight line trend by using method of least squares.

b) Estimate the expected sales in the year 2020.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales (in Lakhs)</td>
<td>7</td>
<td>6</td>
<td>8</td>
<td>8</td>
<td>5</td>
<td>9</td>
<td>13</td>
<td>18</td>
<td>21</td>
<td>20</td>
<td>19</td>
</tr>
</tbody>
</table>

Score: 5 Score, Time: 10 Minutes

Scoring Indicators

Normal Equations ................................................................. (2)
Solution of equations .......................................................... (1)
Trend Equation ................................................................. (1)
Estimation ................................................................. (1)

Learning Outcome

- Estimates the future values by using time series analysis
- Interprets trend lines

Q.13 The exports of food products by a company for the last few years are given in million tonnes.

a) Fit a straight line trend by using method of least squares.

b) Estimate the expected quantity of exports in the year 2022.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Exports</td>
<td>4.8</td>
<td>4.4</td>
<td>4.6</td>
<td>4.2</td>
<td>4.1</td>
<td>4.4</td>
<td>4.3</td>
<td>4.1</td>
<td>4.0</td>
</tr>
</tbody>
</table>

Score: 5 Score, Time: 10 Minutes
Trend Equation ........................................................................................................... (1)
Estimation .................................................................................................................. (1)

**Learning Outcome**

- Estimates the future values by using time series analysis

**Q.14** The imports of fire goods by a company in different years are given below. Obtain the three yearly moving averages

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Imports</td>
<td>6</td>
<td>7</td>
<td>6</td>
<td>9</td>
<td>8</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>14</td>
<td>18</td>
</tr>
</tbody>
</table>

Score: 3 Score, Time: 6 Minutes

**Scoring Indicators:**

Moving Averages ........................................................................................................... (3)

**Learning Outcome**

- Estimates the future values by using time series analysis

**Q.14** The profits of a company in different years are given in the table shown below. Obtain the four yearly moving averages of the data.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit (in Lakhs)</td>
<td>11.1</td>
<td>11</td>
<td>10.8</td>
<td>11.6</td>
<td>12.4</td>
<td>14.2</td>
<td>15.2</td>
<td>14.8</td>
<td>15.6</td>
<td>14.7</td>
<td>15.2</td>
</tr>
</tbody>
</table>

Score: 4 Score, Time: 8 Minutes

**Scoring Indicators**

Moving Averages non centered .................................................................................. (2)
Moving Averages centered ......................................................................................... (2)

**Learning Outcome**

- Estimates the future values by using time series analysis

**Q.15** Obtain the trend values for the following time series by using the two yearly moving averages.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>57</td>
<td>52</td>
<td>51</td>
<td>48</td>
<td>40</td>
<td>51</td>
<td>42</td>
<td>31</td>
<td>33</td>
</tr>
</tbody>
</table>

Score: 3 Score, Time: 6 Minutes

**Scoring Indicators**

Moving Averages non centered .................................................................................. (2)
Moving Averages centered ......................................................................................... (1)
Learning Outcome

- Estimates the future values by using time series analysis

Q.16 The actual values and their moving averages are given in the following table. Find the values for x, y and z.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>14</td>
<td>16</td>
<td>x</td>
<td>18</td>
<td>17</td>
<td>25</td>
<td>23</td>
<td>21</td>
<td>z</td>
</tr>
<tr>
<td>3 yearly moving averages</td>
<td>-</td>
<td>15.33</td>
<td>y</td>
<td>17</td>
<td>20</td>
<td>21.67</td>
<td>23</td>
<td>23.67</td>
<td>-</td>
</tr>
</tbody>
</table>

Score : 3 Score, Time : 8 Minutes

Scoring Indicators

x ................................................................................................................. (1)
y ................................................................................................................. (1)
z ................................................................................................................. (1)

Learning Outcome

- Estimates the future values by using time series analysis

Q.17 The following table was prepared for fitting a trend line using the method of least squares

<table>
<thead>
<tr>
<th>Year (x)</th>
<th>No. of deaths (y)</th>
<th>U =X-2006</th>
<th>u²</th>
<th>uy</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>4</td>
<td>-2</td>
<td>4</td>
<td>-8</td>
</tr>
<tr>
<td>2005</td>
<td>7</td>
<td>-1</td>
<td>1</td>
<td>-7</td>
</tr>
<tr>
<td>2006</td>
<td>11</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2007</td>
<td>13</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2008</td>
<td>17</td>
<td>2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Total</td>
<td>52</td>
<td>0</td>
<td>10</td>
<td>32</td>
</tr>
</tbody>
</table>

(a) Obtain Trend line

(b) Estimate the number of deaths in 2009 (Hint. Equation of Trend line is Y=a+bu)

Score : 4, Time : 8 Minutes

Scoring Indicators

Normal equations ......................................................................................... (1)
Finding the values of ‘a’ and ‘b’ .................................................................. (1)
Trend equation ............................................................................................. (1)
Estimated number of deaths ........................................................................ (1)
Learning Outcome

- Explains the concept of Index Numbers

Q.1 The price relative of a commodity A is 125 and the base year price is 50. Find the current year price.

Score: 2, Time: 3 mts.

⚠️ Scoring Indicators

- Formula of price relative ............................................................... (1)
- \( P_1 = 62.5 \) .................................................................................. (1)

Learning Outcome

- Explains different kinds of simple and weighted index numbers.

Q.2 The price of different food grains in two different years are given below. Construct an index number.

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Production in 2004</th>
<th>Production in 2005</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>15</td>
<td>18</td>
</tr>
<tr>
<td>B</td>
<td>18</td>
<td>22</td>
</tr>
<tr>
<td>C</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>D</td>
<td>15</td>
<td>18</td>
</tr>
</tbody>
</table>

Score: 4, Time: 7 mts.

⚠️ Scoring Indicators

- Formula (either aggregate or average relative method) ........................................... (1)
- Calculation of index number. .................................................................................. (1)
- Interpretation ......................................................................................................... (1)
Learning Outcome

- Identifies and uses consumer price index number.

Q.3 Match the following

A
1. Ideal Index
2. Consumer price Index
3. Current year quantity
4. \( \frac{\sum P_i}{\sum P_0} \times 100 \)

B
Laspeyre's
Simple Index
Fisher's Index
Paasche's Index
Price relative

Score: 2, Time: 2 mts.

Scoring Indicators
Correct matching.
1. \( \rightarrow (3) \); 2. \( \rightarrow (1) \); 3. \( \rightarrow (4) \); 4. \( \rightarrow (2) \) ......................................................... \( \frac{1}{2} \) each

Learning Outcome

- Constructs different kinds of simple and weighted index numbers.

Q.4 Calculate (a) Laspeyre's Index number (b) Fisher's Index number for the following data

<table>
<thead>
<tr>
<th>Base Year</th>
<th>Commodity</th>
<th>Quantity</th>
<th>Price</th>
<th>Current Year</th>
<th>Quantity</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10.0</td>
<td>0.80</td>
<td>11.0</td>
<td>0.70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>8.0</td>
<td>0.85</td>
<td>9.0</td>
<td>0.90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>5.0</td>
<td>1.30</td>
<td>5.5</td>
<td>0.80</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Score: 5, Time 9 mnts

Scoring Indicators
Calculation of \( \sum p_0q_0, \sum p_1q_0, \sum p_1q_0, \sum p_1q_1 \) .............................................. (2)
Laspeyre's index number formula + calculation ......................................................... (1)
Laspyre's Index Number = 85.4
Paasche's index number formula + calculation ......................................................... (1)
Paasche's index number = 85.6
Fishers index number formula + calculation ............................................................ (1)
Fishers index number = 85.5
Learning Outcome

- Constructs different kinds of simple and weighted index numbers.

Q.5 Complete the missing values (items) in the given table for calculating Fisher's ideal index number.

<table>
<thead>
<tr>
<th>Commodity</th>
<th>1995</th>
<th>2005</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( p_0 )</td>
<td>( q_0 )</td>
</tr>
<tr>
<td>A</td>
<td>12</td>
<td>100</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>200</td>
</tr>
<tr>
<td>C</td>
<td>8</td>
<td>120</td>
</tr>
<tr>
<td>D</td>
<td>20</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\text{Laspeyer's Index} = \frac{\sum p_0 q_0}{\sum p_1 q_0} = 136.5
\]

\[
\text{Paasche's Index} = \frac{\sum p_0 q_1}{\sum p_1 q_1}
\]

\[
\text{Fishes Index} = \sqrt{136.5 \times 138.3} = 137.4
\]

Score: 6, Time: 12 mts.

**Scoring Indicators**

\[
\frac{5680}{4160} \times 100 = 136.5
\]

\[
\frac{6360}{4600} \times 100 = 138.3
\]

\[
\sqrt{136.5 \times 138.3} = 137.4
\]

Missing values:

: 800, 960, 1200
: 1440, 1200, 1000 (4600)
: 2000, 800, 1440, 1440 (5680)
: 2400, 960, 1800, 1200
**Learning Outcome**

- Constructs different kinds of simple and weighted index numbers.

**Q.6** Laspeyre’s Index number for a group of commodities is 121 and Paasche’s Index number for the same group is 144. What will be Fisher’s Index number?

Score: 1, Time: 2 mts.

**Scoring Indicators**

Finds the realtionship

\[ F = \sqrt{L \times P} = \sqrt{121 \times 144} = 11'12 = 132 \] ............................ (1)

**Learning Outcome**

- Identifies and uses consumer price index number.

**Q.7** Different classes of people or same classes of people living at different places consume different types of commodities

a. Name the most suitable Index number for getting an exact idea of this change.

b. Suggest the formula

c. What are the other uses of this index number?

Score: 4, Time: 10 mts.

**Scoring Indicators**

a. Consumer price index number (CPI) ................................................................. (1)

Identifies, calculates

\[ \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100 \]

b. \[ \frac{Total \ expenditure \ in \ current \ year}{Total \ Expenditure \ in \ base \ year} \times 100 \] ............................ (1)

c. Any 4 Uses ...................................................................................................... (2)

**Learning Outcome**

- Identifies and uses consumer price index number.

**Q.8** From the following figures prepare cost of living index numbers by Aggregate expenditure method.
<table>
<thead>
<tr>
<th>Articles</th>
<th>Quantity</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>consumed in</td>
<td>Base year</td>
</tr>
<tr>
<td></td>
<td>Base Year</td>
<td></td>
</tr>
<tr>
<td>Wheat</td>
<td>4 (quintal)</td>
<td>50</td>
</tr>
<tr>
<td>Rice</td>
<td>1 (quintal)</td>
<td>80</td>
</tr>
<tr>
<td>Grain</td>
<td>1 (quintal)</td>
<td>40</td>
</tr>
<tr>
<td>Pulses</td>
<td>2 (quintal)</td>
<td>80</td>
</tr>
<tr>
<td>Ghee</td>
<td>50 (kilogram)</td>
<td>10</td>
</tr>
<tr>
<td>Sugar</td>
<td>50 (kilogram)</td>
<td>1</td>
</tr>
<tr>
<td>Firewood</td>
<td>5 (quintal)</td>
<td>10</td>
</tr>
<tr>
<td>Home Rent</td>
<td>1 (House)</td>
<td>50</td>
</tr>
</tbody>
</table>

Score: 4, Time: 8 mts.

⚠️ Scoring Indicators

- Identification of $p_1$, $p_0$, and $q_0$.  
- Calculation of $\sum p_1 q_0$ and $\sum p_0 q_0$.  

Cost of living Index $= \frac{\sum p_1 q_0}{\sum p_0 q_0} = \frac{2555}{1130} \times 100 = 226.1$  

---

**Learning Outcome**

- Constructs different kinds of simple and weighted index numbers.

**Q:9**

(a) If the retail price of rice in the year 2011 was Rs.22/- and that of the year 2014 was Rs.38/-.. Calculate the price relative index for rice.

(b) For the data given below, simple index number for the year 2005 by taking 2000 as base year is 125. Current year prices are 60, 50, Y, 120, 100. Sum of base year prices is 550. Find the values of Y.

Score 4, Time: 8 mts.

⚠️ Scoring Indicators

- Price relative of Rice.  
- Simple aggregative index formula.  

Finding $\sum p_0$ and $\sum p_1$.  
Calculate value of $y = 70$.  

---
Learning Outcome

- Constructs different kinds of simple and weighted index numbers.

Q:10 For the data given below simple index number for the year 2005 by taking 2000 as base year is 125

<table>
<thead>
<tr>
<th>Commodity</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price in 2000 ( (P_0) )</td>
<td>40</td>
<td>X</td>
<td>50</td>
<td>80</td>
</tr>
<tr>
<td>Price in 2005 ( (P_1) )</td>
<td>60</td>
<td>40</td>
<td>Y</td>
<td>80</td>
</tr>
</tbody>
</table>

Also \( \sum p_0 = 200 \) Find the values of \( X \) and \( Y \)

Score: 4, Time: 7 mts.

⚠️ Scoring Indicators

Simple aggregative index formula ......................................................... (1)
Finding \( \sum p_1 \) .................................................................................. (1)
Calculate value of \( X \) and \( Y \) ............................................................ (1)